III. CONTRIBUTION TO STOCHASTIC GW BACKGROUND

In this section we investigate the halo mergers as sources of stochastic GW background.

It is generally believed that almost all galaxies harbor one or more supermassive black holes (SMBHs). When two galaxies merge, the SMBHs in the host galaxies will form a binary system. Gravitational wave will be radiated in the subsequent evolution. Here, we investigate the contribution to stochastic GW background due to the galaxy mergers.

When two galaxies are rotating around each other, the frequency of GWs is approximately $f = \omega/2\pi = \sqrt{GM/d^3}/2\pi \approx 10^{-17} - 10^{-16}$ Hz for a Milky Way-like galaxy ($M \approx 10^{12} M_\odot$, $d \approx 30kpc$), which is much lower than the sensitive window of current and next generation GW detectors. Hence, we only consider the GW signals from mergers of SMBHs, of which the frequency is $f \approx \sqrt{GM/(2R_s)^3}/2\pi \approx 10^{-3}$Hz where M is taken to be $10^6 M_\odot$ with $R_s^2 = 2GM/c^2$. The signals from SMBHs are capable of detecting by the space-based Laser Interferometer Space Antenna (LISA).

Like Ω_m and Ω_Λ, we define the density parameter of GW background

$$\Omega_{GW} = \frac{1}{\rho_c} \frac{d\rho_{GW}}{d \ln f}. \tag{7}$$

B. Allen (1996) showed the relationship between gravitational-wave power spectral density $S_h(f)$ and $\Omega_{GW}(f)$

$$S_h(f) = \frac{3H_0^2}{2\pi} \frac{\Omega_{GW}(f)}{f^3}. \tag{8}$$

Further, E. Phinney (2001) and X.-J. Zhu et al (2013) derived the equation for calculating Ω_{GW} using the distribution of black hole binary systems

$$\Omega_{GW}(f) = \frac{f}{\rho_c H_0 c^2} \int dz \frac{\mathcal{R}(z)}{(1+z)\sqrt{\Omega_m (1+z)^3 + \Omega_\Lambda}} \frac{dE_{GW}}{df}(f_s) \tag{9}$$

where $\mathcal{R}(z)$ is the merger rate per unit comoving volume per unit redshift interval per time and $f_s = (1+z)f$. In this paper, we use the power spectrum given by P. Ajith et al (2009) and X.-H. Zhu et al (2011) for the inspiral, merger and ringdown phases

$$\frac{dE}{df} = \frac{(G\pi)^{2/3} M_c^{5/3}}{3} \begin{cases} f^{-1/3} & f < f_{merg} \\ f_{merg}^{-1} f^{2/3} & f_{merg} \le f < f_{ring} \\ f_{merg}^{-1} f_{ring}^{-4/3} \left[\frac{f}{1 + \left(\frac{f - f_{ring}}{\sigma_f} \right)^2} \right]^2 & f_{ring} \le f < f_{cut} \end{cases} \tag{10}$$

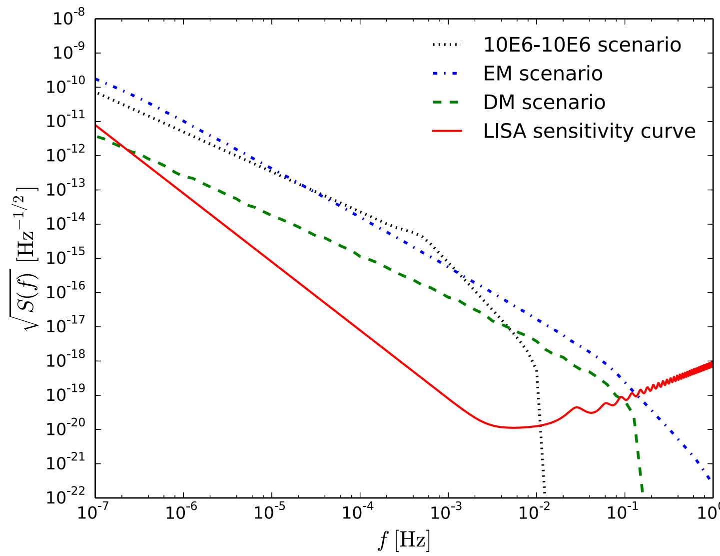

FIG. 2. GW power spectra v.s. LISA sensitivity

where $M_c = [M_1 M_2 (M_1 + M_2)^{-1/3}]^{3/5}$ is called as chirp mass and M_1, M_2 are SMBH masses. The characteristic frequencies f_{merg}, f_{ring} and f_{cut} can be written in a universal function of symmetric mass ratio $\theta = M_1 M_2 / M^2$ and total mass $M = M_1 + M_2$, such as $(a\theta^2 + b\theta + c)/(\pi M)$. Relevant parameters (a, b, c) for f_{merg}, f_{ring}, f_{cut} can be found in the table 1 of P. Ajith et al (2009).

At first, we consider a conservative case that $M_1 = M_2 = M_{SMBH} = 10^6 M_\odot$ (10E6-10E6 scenario). The corresponding spectral parameters are $(f_{merg}, f_{ring}, \sigma_f, f_{cut}) = (4.04 \times 10^{-3}, 8.07 \times 10^{-3}, 2.37 \times 10^{-3}, 1.15 \times 10^{-2})$Hz. Consider the merger rate of galactic halos, we estimate

$$\mathcal{R}(z) = \frac{1}{2} \int dM \frac{dN}{dM} \frac{P(M, z)}{t_{age}}, \tag{11}$$

here the factor 1/2 counts for the binary SMBH systems. To compare the signal strength and the sensitivity of LISA, we plot the source spectral density $\sqrt{S_h(f)}$, as can be found in figure 7 (dotted line). As we can find in this figure, the GW background produced by galaxy-merger-induced SMBH mergers is supposed to be detected by the space based GW detector LISA.

Now we consider one more realistic scenario that both M_1 and M_2 can vary with redshift. At first we establish the relation between the mass of SMBH M_{SMBH} in a galaxy center and the mass of the host galaxy. Roughly speaking, M_{SMBH} is proportional to M_{spher}, the mass of spheroid, and the ratio is $M_{SMBH}/M_{spher} \approx 0.001 - 0.006$ (Kormendy & Richstone 1995; Magorrian et al. 1998; Merritt & Ferrarese 2001). Also, M. Enoki et al (2004) showed that in a no random collision model the SMBH of $10^9 M_\odot$ corresponds to a massive galaxy of $10^{12} M_\odot$. Hence, in this paper we use the mass ratio $M_{SMBH}/M_g = 0.001$. Considering the merger of SMBHs with different masses, equation 23 can be rewritten as

$$\Omega_{GW}(f) = \frac{f}{\rho_c H_0 c^2} \int \frac{\mathcal{N}(M_1^h, M_2^h, z)}{(1+z)\sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda}} \frac{dE_{GW}(M_1, M_2, f_s)}{df} |_{(M_1,M_2)=10^{-4}(M_1^h,M_2^h)} dM_1^h dM_2^h dz, \tag{12}$$

where $\mathcal{N}(M_1^h, M_2^h, z)$ represents the number of mergers per unit time per unit comoving cosmic volume per unit intervals of redshift z, halo masses M_1^h and M_2^h.

Here we consider two situations. The first one is equal-mass (EM) scenario that $M_1 = M_2 = 10^{-4}M^h$. In this case $\mathcal{N}(M_1^h, M_2^h) = \frac{1}{2}\frac{dN}{dM^h}\frac{P(M^h,z)}{t_{age}}|_{M^h=M_1^h=M_2^h}$ and $\frac{dE_{GW}}{df} = \frac{dE_{GW}}{df}(M_1, M_2, z)|_{M_1=M_2=10^{-4}M^h}$. Also, the integrand in equation (26) will be integrated over M^h and z. The corresponding spectral density is shown as the dash-dotted line in figure 7. If M_1 and M_2 can vary independently and the mergers can take place for SMBHs with different masses (DM scenario), using the mass function of halos, $\mathcal{N}$ can be expressed as

$$\mathcal{N}(M_1^h, M_2^h, z) = \frac{1}{2}\frac{dN}{dM_1^h}\frac{dN}{dM_2^h}\frac{P(M_1^h, z)P(M_2^h, z)}{t_{age}} \left(\int \frac{dN}{dM^h} P(M^h, z) dM^h\right)^{-1}. \tag{13}$$

Combining eqn 24, 26 and 27, we obtain the $\Omega_{GW}(f)$ and the spectral density $\sqrt{S_{GW}(f)}$ (dashed line in figure 7).

As for EM and DM scenarios, since the mass of SMBH can be smaller than $10^6 M_\odot$ especially for early-type galaxies, the spectra truncate at higher frequencies. Moreover, in the DM scenario, heavier SMBHs have a larger probability to merge with SMBHs of relatively lower masses, due to the population distribution, therefore we obtain a spectrum lower than EM scenario where heavier SMBHs only merge with SMBHs of equal mass. In any case, the stochastic background from galaxy mergers is a promising GW source for LISA.