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NEUTRINO OSCILLATIONS IN MATTER AND MASS HIERARCHY

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ABSTRACT

In this paper, we formulize the matter effect in two-flavor neutrino oscillations and investigate the applicability of solving missing solar neutrino problem. Besides, the matter effect can also be used to determine the mass hierarchies, e.g. sign$\{\Delta m^2_{21}\}$ and sign$\{\Delta m^2_{31}\}$ which are crucial in the flavor discipline in lepton sector. We show that for the solar neutrino case, normal hierarchy $\Delta m^2_{21} > 0$ is unambiguously and robustly validated. As for sign$\{\Delta m_{31}\}$, we find that matter effect in three-flavor oscillations enables us to distinguish the normal hierarchy and inverted hierarchy. The performances of proposed PINGU and long-base accelerator/reactor experiments, such as NOνA and T2K, are reviewed.

Keywords: Neutrino Physics; Neutrino Experiments

1. INTRODUCTION

As an elementary constituent of weak interactions and particle physics, neutrinos continuously increase peoples understanding of fundamental physics, and neutrino physics has become one of the most fertile frontiers over the last few decades. Neutrino oscillations, firstly predicted by Bruno Pontecorvo in 1957 (Pontecorvo 1957) and further developed by Maki, Nakagawa, Sakata (Maki et al. 1962) and Pontecorvo (Pontecorvo 1968) in 1960s, successfully resolved the solar neutrino problem. Neutrino oscillations reveal the quantum-mechanical nature of neutrinos where the three eigenstates in weak interactions ν_e, ν_μ and ν_τ can be represented as the linear combinations of the free-particle eigenstates ν_1, ν_2 and ν_3. In the framework of Stand Model, the neutrino flavor transition probability depends on the differences of squares of the neutrino masses $\Delta m^2_{ij} = m^2_i - m^2_j$, the traveling distance as well as three mixing angles θ_{ij} and one CP phase δ_{CP} in the unitary PMNS matrix named after Pontecorvo, Maki, Nakagawa and Sakata. The unitarity of the PMNS matrix implies that we can rewrite it as the multiplication of three matrices representing the atmospheric oscillation, cross oscillation and solar oscillation using three rotation angles and one phase factor. Recent experiments have constrained the maxing angles, Δm^2_{12} and $|\Delta m^2_{13}|$ to the percent level (Gonzalez-Garcia et al. 2012; Fogli et al. 2012). Considering the discovery of non-zero mixing angle θ_{13} (Abe et al. 2012; An et al. 2012; Ahn et al. 2012), the major intriguing questions in neutrino experiments are the unambiguous determination of the mass hierarchy (the sign of Δm^2_{13}) and the CP phase δ_{CP}.

Without the presence of matter on the trajectory, neutrinos are supposed to oscillate intrinsically due to flavor mixing of mass eigenstate ν_1, ν_2 and ν_3. In this case, the transition probabilities between weak interaction flavors ν_e, ν_μ and ν_τ depend only on the absolute values of the mass square differences Δm^2_{ij}. To determine the mass hierarchy, the order of m_i, matter effects in the sun for solar neutrinos and in the earth for atmospheric neutrinos and accelerator/reactor neutrinos break the protection of $\sin^2 \frac{\Delta m^2_{ij} L}{2E}$ and make the transition probabilities sensitive to the sign of Δm^2_{ij}. Therefore, the matter effect plays a significant role in the experiments dedicated to determine the mass hierarchies.

In this paper, we are going to discuss the neutrino oscillations in vacuum and the well-known MSW effect which describes the oscillations in matter in section 2. In section 3, the applications of matter effects are investigated, such as solar neutrino problem and the mass hierarchy Δm^2_{21}. A brief review of the proposed experiments on sign$\{\Delta m^2_{31}\}$ is also presented in section 3. Summary is given in section 4.

2. NEUTRINO OSCILLATIONS IN MATTER

Since the eigenstates of weak interactions (ν_e, ν_μ and ν_τ) are not eigenstates of the free-particle Hamiltonian (ν_1, ν_2 and ν_3), the time-dependent solution to the Schrodinger equation contains the time-varying factor $e^{ip_i x}$ for each eigenstate ν_i, $(i = 1,\ 2,\ 3)$, which results in the oscillation in the transition probability. In this section, let's focus on the neutrino oscillations inside the vacuum and the matter and derive the transition probabilities that will be applied in the section 3 to the neutrino experiments.

2.1. *Neutrino oscillations in vacuum*

In terms of $|\nu_i\rangle$, a neutrino state $|\nu_\alpha\rangle$ can be written in the form $|\nu_\alpha\rangle = \sum_i U_{\alpha i}|\nu_i\rangle$. After a time t, the state $|\nu_\alpha\rangle$ becomes $|\nu_\alpha\rangle_t = \sum_i U_{\alpha i} e^{-iE_i t}|\nu_i\rangle$ and the transition probability to the state $|\nu_\beta\rangle$ is

$$P_{\alpha\to\beta}(L) = |\langle\nu_\beta|\nu_\alpha\rangle_t|^2 = \sum_i |U_{\alpha i}|^2|U_{\beta i}|^2 + \sum_{i\neq j} U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j}\exp\left(i\frac{L}{2E}\Delta m^2_{ji}\right) \quad (1)$$

where $L = ct$ is the distance traveled by the neutrino and $\Delta m^2_{ji} = m^2_j - m^2_i$. To derive Eq. 1 we use the fact that $|\boldsymbol{p}| \gg m_i$ and the approximation $E_i = \sqrt{p^2+m_i^2} \simeq p + m_i^2/2E$. For two-flavor oscillations, such as $\nu_e - \nu_\mu$ oscillations in solar neutrinos, the unitary matrix U can be written as the 2-dimensional rotation matrix with the angle θ. In this case, the survival probability of ν_e can be written explicitly as

$$\begin{aligned} P_{e\to e}(L) &= 1 - \sin^2(2\theta)\sin^2\left(\frac{\Delta m^2_{12}L}{4E_\nu}\right) \\ &= 1 - \sin^2(2\theta)\sin^2\left(\pi\frac{L}{\lambda_{\rm osc}}\right) \end{aligned} \quad (2)$$

where the wavelength of oscillation is given by $\lambda_{\rm osc} = 2.47\text{km}(E_\nu/1\ \text{GeV})(\Delta m^2_{12}/\text{eV}^2)$. In general, the oscillation probability of 3-flavor neutrinos can be obtained by simplifying Eq. 1 using the unitarity of matrix U. We have

$$P_{\alpha\to\beta}(L) = \delta_{\alpha\beta} - 4\sum_{i>j}\mathcal{R}(U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j})\sin^2\left(\frac{\Delta m^2_{ij}L}{4E_\nu}\right) + 2\sum_{i>j}\mathcal{I}(U^*_{\alpha i}U_{\beta i}U_{\alpha j}U^*_{\beta j})\sin^2\left(\frac{\Delta m^2_{ij}L}{4E_\nu}\right) \quad (3)$$

where $\mathcal{R}$ and $\mathcal{I}$ return the real and imaginary part of a complex number, respectively.

So far, we have obtained the transition probabilities for neutrino oscillations in vacuum. One common feature is that the probability depends only on the square of Δm^2_{ji}, which obstacles the determination of mass hierarchy, say the order of m_i. To solve this problem, we need to measure the oscillation of neutrinos inside matter, e.g. the earth itself or the interior of the sun for solar neutrinos.

2.2. *Neutrino oscillations in matter: MSW effect*

Now, we consider how matter effects the oscillation probabilities in Eqs. 2 and 3. While propagating inside matter, neutrinos may interact with electrons through neutral current and/or charged current interactions and

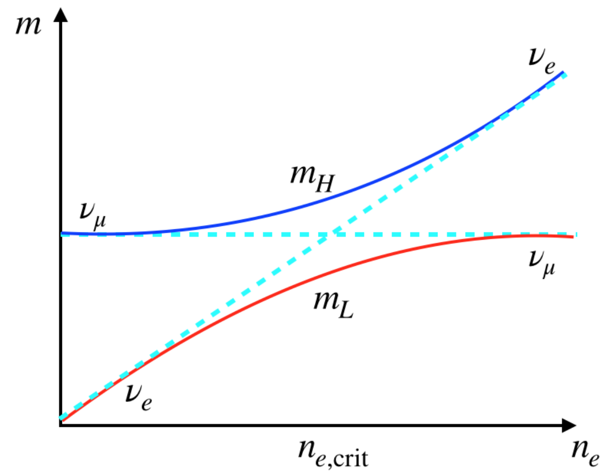

Figure 1. Evolutions of m_H (blue line) and m_L (red line) as n_e changes.

an external potential due to these interactions can introduce an external phase to the oscillation probability, as pointed by Mikheyev, Smirnov and Wolfenstein (Mikheyev & Smirnov 1985; Wolfenstein 1978). Here, to simplify the calculation, let's follow the history of solar-neutrino puzzles considering the mixing between ν_e and ν_μ with the presence of matter.

Firstly, we write the $\nu_e - \nu_\mu$ oscillations in vacuum (Eq. 2) in the form of Schrodinger equation. It's easy to verify that Eq. 2 can be derived from the equation $\frac{d}{dt}(|\nu_e\rangle, |\nu_\mu\rangle)^T = \hat{H}_{\rm vac}(|\nu_e\rangle, |\nu_\mu\rangle)^T$, where the vacuum Hamiltonian can be written in terms of Pauli matrices as

$$\hat{H}_{\rm vac} = \frac{\omega}{2}(\sigma_1\sin 2\theta - \sigma_3\cos 2\theta), \quad (4)$$

where $\omega = \frac{\Delta m^2}{2E_\nu}$. In this paper, the term $\Delta m^2 = \Delta m^2_{21} > 0$ follows the normal hierarchy in two flavor oscillations. The effective perturbation to the vacuum Hamiltonian from neutrino-electron interactions is given by (Mikheyev & Smirnov 1985)

$$\hat{H}_{\rm int} = \frac{G_F}{\sqrt{2}}n_e(x)\sigma_3 = \Omega\sigma_3/2,\ \text{with}\ \Omega = \sqrt{2}G_F n_e(x) \quad (5)$$

which can be deduced from the weak interaction amplitude $M_{fi} = \frac{G_F}{\sqrt{2}}\bar{\nu}_e\gamma_\mu(1-\gamma^5)\nu_e\bar{e}\gamma^\mu(1-\gamma^5)e \simeq \langle f|\hat{H}_{\rm int}|i\rangle$. Here the electron density can be a function of position. Before trying to solve the equation of motion. It's worthy discussing two limits. If the interaction term is much larger than the vacuum hamiltonian, e.g. $G_F n_e \gg \omega$, the Hamiltonian is auto-diagonalized, the neutrinos are frozen to their initial flavor state in the propagation. Otherwise, if $G_F n_e \ll \omega$, the oscillation reduces to the vacuum case.

It's a little tricky of solving the equation of motion. Here I present one diagonalize scenario. Define the position dependent rotation matrix $U_X(\tilde{\theta}(x))$ such that $H' = U_X^{-1}(\hat{H}_{\rm vac} + \hat{H}_{\rm int})U_X$ is diagonalized. We can rewrite $(|\nu_e\rangle\,,\ |\nu_\mu\rangle)$ in the basis of the new eigenstates of H' notating as $(|\nu_L\rangle\,,\ |\nu_H\rangle)$,

$$(|\nu_e\rangle\,,\ |\nu_\mu\rangle)^T = U_X(\tilde{\theta}(x))(|\nu_L\rangle\,,\ |\nu_H\rangle)^T. \qquad (6)$$

The next step is to set the off-diagonalized components of H' to zero. From the two resulting equations, we can solve $\cos 2\tilde{\theta}(x)$

$$\cos 2\tilde{\theta}(x) = \frac{\cos 2\theta - \Omega/\omega}{\sqrt{\frac{\Omega^2}{\omega^2} + 1 - 2\frac{\Omega}{\omega}\cos 2\theta}}. \qquad (7)$$

The energy eigenvalues are

$$m^2_{L,H} = \sqrt{2}E_\nu G_F n_e \mp \frac{1}{2}\sqrt{(2\sqrt{2}E_\nu G_F n_e - \Delta m^2 \cos 2\theta)^2 + (\Delta m^2)^2 \sin^2 2\theta}. \qquad (8)$$

Eqs 7 and 8 allow resonance in the oscillation. By simply rewriting the denominator of Eq. 7, we see that resonance occurs if $\Omega/\omega = \cos 2\theta$ and we define the critical density

$$n_{e,\rm crit} = \frac{\Delta m^2}{2\sqrt{2}G_F E_\nu}\cos 2\theta. \qquad (9)$$

If electron neutrinos are produced in the region $n_e \gg n_{e,\rm crit}$ where $\tilde{\theta} \simeq \pi/2$ and $|\nu_2\rangle \simeq |\nu_H\rangle$, the electron neutrinos will follow the heavy-mass trajectory, as shown in the Fig. 1. Once n_e changes adiabatically to the case $n_e \ll n_{e,\rm crit}$ where $|\nu_H\rangle \simeq |\nu_\mu\rangle$, therefore almost all electrons are converted to muon neutrinos.

To justify the adiabatic condition, we set one criterion that if the energy gap δE times the transition time δt is much larger than $\hbar$ we say the process is adiabatic. The energy gap satisfies $\delta E \le \frac{m_H^2 - m_L^2}{2E}|_{n_e = n_{e,\rm crit}} = \frac{\Delta m^2 \sin 2\theta}{2E}$. δt is related to the electron density gradient dn/dr. After some calculations, we obtain the adiabatic condition

$$\frac{1}{n_e}\frac{dn_e}{dr} \ll \frac{\Delta m^2}{2E_\nu}\frac{\sin^2 2\theta}{\cos 2\theta}. \qquad (10)$$

Then the probability for $\nu_e \to \nu_e$ when n_e changes from extreme dense state to extreme thin state adiabatically can be calculated by averaging the time-varying part, e.g. Bethe (1986),

$$P_{e\to e} = \frac{1}{2}(1 + \cos 2\theta \cos 2\tilde{\theta}), \qquad (11)$$

where $\tilde{\theta}$ is the mixing angle at the initial point.

In general, if the adiabatic condition Eq. 10 is not satisfied, the state of the upper branch may cross the asymptotic line and cause level mixing. The probability of branch crossing is given by $P_f = \exp(-\frac{\pi}{2}\gamma)$ (Parke 1986; Haxton 1986) where

$$\gamma = \frac{\Delta m^2 \sin^2 2\theta}{2E_\nu \cos 2\theta (1/n_e)(dn_e/dr)}. \qquad (12)$$

With the transiting probability P_f, the time-averaging survival probability is modified to

$$P_{e\to e} = \frac{1}{2} + \left(\frac{1}{2} - P_f\right)\cos 2\theta \cos 2\tilde{\theta}. \qquad (13)$$

These equation will be used in section 3 to fit the solar neutrino oscillation data.

3. DETERMINATION OF MASS HIERARCHY

3.1. *Solar neutrino oscillations:* $\Delta m^2_{21} > 0$

The nuclear fusions inside the sun, especially ^8B can produce copious quantities of neutrinos in the energy range $0.1-10$MeV. In April 1968, Davis Jr et al. (1968) found that the upper bound of solar electron neutrino flux is around the 1/3 of the prediction from standard solar model (Bahcall et al. 1968). This contradiction remains a puzzle around 30 years. Here, with the MSW theory in neutrino oscillations, I'll try to explain this missing-neutrino problem and to fit the solar neutrino flux.

To find how electron neutrinos are converted to muon neutrinos, I solve the Schrodinger equation numerically along the propagation inside the sun. Using the combined wave function $\psi = (\phi_{\nu_e},\ \phi_{\nu_\mu})^T$, we obtain the dimensionless equation governing the neutrino oscillation inside the sun

$$i\partial_x \psi = \frac{R_s \omega}{2}\left[\left(\frac{\Omega}{\omega} - \cos 2\theta_\nu\right)\sigma_3 + \sin 2\theta_\nu \sigma_1\right]\psi, \qquad (14)$$

where $x = r/R_s$ and $R_s \simeq 3.5 \times 10^{24}$ GeV^{-1} is the solar radius. The electron neutrino survival probability $P_e(x) = |\phi_{\nu_e}|^2$ and the muon neutrino production probability $P_\mu(x) = |\phi_{\nu_\mu}|^2$ can be obtained numerically using the initial condition $\phi_{\nu_e}(0) = 1$ and $\phi_{\nu_\mu}(0) = 0$ and the solar electron density profile $n_e(x) \simeq 10^{-13-4.3x}$ GeV^3. Noting that the solutions depend on $\omega = \frac{\Delta m^2}{2E_\nu}$, in figure 2 I present the numerical results for electron neutrino probabilities (red lines) and muon neutrino probabilities (green lines) when the factor ω is assumed to be 10^{-18} GeV (left), 10^{-19} GeV (middle) and 10^{-20} GeV (right). As we can see the red lines in these figure, the matter effect in neutrino oscillations perfectly resolves the missing solar neutrino problem when using the normal mass hierarchy $\Delta m^2_{21} = m_2^2 - m_1^2 > 0$ and $\omega > 0$. Meanwhile, for a lower ω, the term Ω/ω becomes dominant in Eq. 14 and the intrinsic oscillations due to $\cos 2\theta$ are suppressed. This effect is verified by the three figure from left to right in Fig. 2. Hence, electron neutrinos

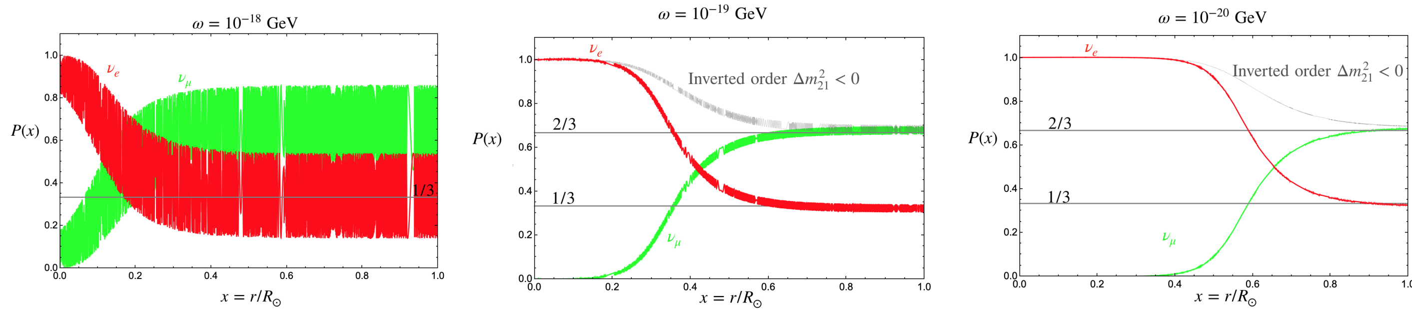

Figure 2. Electron neutrino probabilities (red lines) and muon neutrino probabilities (green lines) when the factor ω is assumed to be 10^{-18} GeV (left), 10^{-19} GeV (middle) and 10^{-20} GeV (right) assuming a normal mass hierarchy. The electron neutrino probabilities for inverted mass hierarchy are illustrated as thin gray lines.

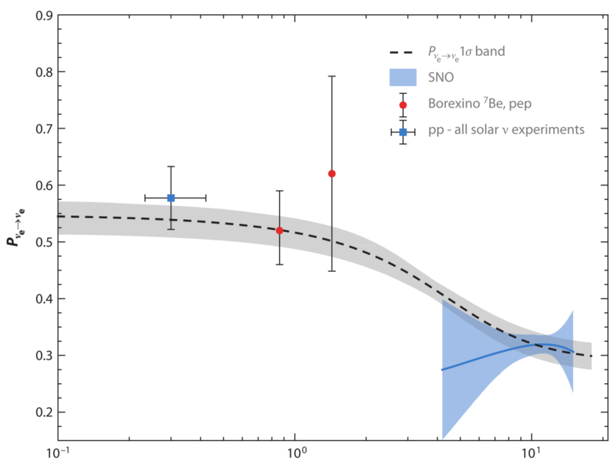

Figure 3. Time averaging survival probabilities $P_{e\to e}$ at various neutrino energies. The gray area takes shows the current uncertainties in Δm_{21}^2 and $\sin^2 2\theta_{21}$. Figure is taken from Haxton et al. (2013)

and muon neutrinos are more distinguishable as E_ν increases (ω descreases).

Typically, here we show that the matter effect in neutrino oscillations can be used to distinguish the normal hierarchy and the inverted mass hierarchy, i. e. $\Delta m_{21}^2 = m_2^2 - m_1^2 < 0$ and $\omega < 0$. To see this, we can replace positive ω in Eq. 14 by a negative one $-|\omega|$. The electron neutrino probabilities for the inverted hierarchy assuming $|\omega| = 10^{-19}$ GeV and $|\omega| = 10^{-20}$ GeV are shown in the second and third figures as thin gray lines in Flg. 2. We conclude that inverted hierarchy leads to more electron neutrinos, more specifically nearly twice as much as being supposed from normal mass hierarchy, which violates the upper bound of electron neutrinos observed on the earth.

In recent year, the mixing angle and mass square difference between ν_1 and ν_2 are measured to high precise, $\Delta m_{21}^2 = (7.6\pm0.2)\times10^{-5}$ eV^2 and $\sin^2\theta_{12} = 0.87\pm0.04$. Using these values and the time averaging electron probability Eq. 13, we can fit the measured survival probability at different energies, as illustrated in Fig. 3 (Haxton et al. 2013). The MSW prediction is in good agreement with experimental constraints.

3.2. sign$\{\Delta m_{31}^2\}$

The three flavor neutrino oscillations in matter are discussed by Kuo & Pantaleone (1989); Ohlsson & Snellman (2000). The formulism is more complicated than two-flavor case and here we just show the equations for the wave function $\psi_m = (\phi_{\nu_e},\ \phi_{\nu_\tau},\ \phi_{\nu_\tau})^T$, e.g. $i\partial_x\psi_m = \hat{H}_m\psi$. The Hamiltonian is

$$\hat{H}_m = \Omega R_s \begin{pmatrix} U_{e1}^2 - \frac{1}{3} + \frac{\Delta m_{12}^2+\Delta m_{13}^2}{6E\Omega} & U_{e1}U_{e2} & U_{e1}U_{e3} \\ U_{e2}U_{e1} & U_{e2}^2 - \frac{1}{3} + \frac{\Delta m_{12}^2+\Delta m_{23}^2}{6E\Omega} & U_{e2}U_{e3} \\ U_{e1}U_{e3} & U_{e2}U_{e3} & U_{e3}^2 - \frac{1}{3} + \frac{\Delta m_{13}^2+\Delta m_{23}^2}{6E\Omega} \end{pmatrix} \quad (15)$$

where U_{ei} are the first row components in the PMNS matrix. OctoMiao (2016) has tried to solve the transition probabilities. The parameters used in the calculations are $\Delta m_{12}^2 = 7.6\times10^{-5}$ eV^2, $|\Delta m_{31}^2| \approx |\Delta m_{32}^2| = 2.3\times10^{-3}$ eV^2. The ternary diagrams in Fig. 4 show the neutrino oscillations for the normal mass hierarchy $\Delta m_{31}^2 > 0$ (left diagram) and inverted hierarchy $\Delta m_{31}^2 < 0$ (right diagram) assuming the neutrino energy is $E = 1$ MeV. Similar with the results in two-flavor oscillarions, the oscillating shape changes dramatically when assuming an inverted hierarchy since the oscillating frequency due to matter effect changes.

Experimentally, the matter effect of the earth opens a window of determining the sign of Δm_{31}^2. Here, I review

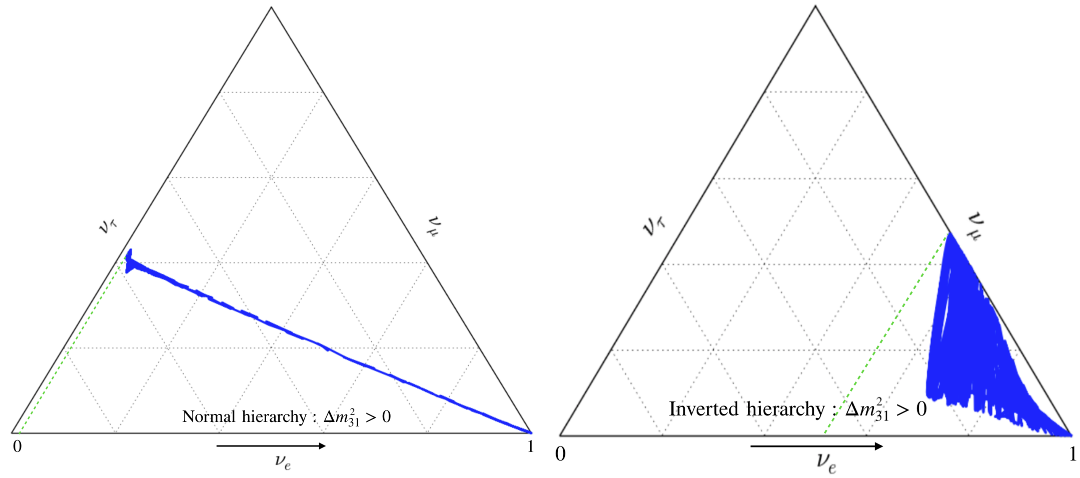

Figure 4. Three flavor oscillations for the normal hierarchy $\Delta m^2_{31} > 0$ (left) and inverted hierarchy $\Delta m^2_{31} < 0$ (right.)

the results from IceCube-PINGU where atmospheric neutrinos are used and compare this method with long-base oscillation experiments. For high-precision measurements, we should take the energy difference Δm^2_{21} in to account. Since the normal hierarchy Δm^2_{31} measure the difference between the squares of heaviest and lightest masses while for the inverted hierarchy, it corresponds to the difference between the lightest and the second heaviest neutrinos. Exactly speaking, changing from normal hierarchy to inverted hierarchy is not simply flipping the sign of Δm^2_{31}. This effect is confirmed through simulations presented by Winter (2013). Hence, in realistic analysis, this can be solved by defining an effective mass square difference $(\Delta m^2_{31})_{\mathrm{eff}}$

$$(\Delta m^2_{31})_{\mathrm{eff}} = \Delta m^2_{31} - \Delta m^2_{21} \times (\cos^2\theta_{12} - \cos\delta \sin\theta_{13} \sin 2\theta_{12} \tan\theta_{23}). \quad (16)$$

This expression entangles Δm^2_{31} with other parameters such as CP phase δ. In simulations, all these parameters should be fitted by minimizing the χ^2 function and these parameters may influence the detector exposure time for a 3σ discovery.

Fig. 5 (Winter 2013) illustrates the fraction of CP phase, $\delta/2\pi$, for which the mass hierarchy can be determined by 3-year PINGU exposure (left figure) with 90% confident level and 8-year exposure at 3σ confident level. The expected performances of long-base accelerator and reactor experiments especially NOνA and T2K in 2020 and 2025 are shown as the green bars. The blue bars represent the combined performances. The different bar groups correspond to different θ_{23} and the mass hierarchies, e.g. NH stands for the normal hierarchy and IH is the inverted hierarchy. For instance, from the first group in the left figure we find that if neutrino eigenstates are described by normal mass hierarchy, θ_{23} is 40 deg and $0 \leq \delta/2\pi \lesssim 0.4$, the three-year PINGU exposure can unambiguously confirm that at the confidence level 90%. Comparing different groups we find that if $\theta_{32} = 50$ deg rather than 40 deg, the PINGU performance will significantly get improved. For example, if $\theta_{32} = 50$ deg, the PINGU alone can reach the full coverage of δ in both NH and IH cases. We can also see that the normal hierarchy can shorten the exposure time required to announce the discovery at 90% and 3σ confidence levels. In the future, with the cooperation of PINGU and accelerator/reactor experiments, it is possible to get a 3σ determination of the Δm^2_{31} hierarchy in the whole range of CP phase.

4. SUMMARY

Matter effect in solar neutrino oscillations can successfully resolve the neutrino missing problem and enable us to determine the hierarchy $\Delta m^2_{21} > 0$. With the discovery of nonzero θ_{31}, the hierarchy of m_2 and m_3 is proposed to be discovered using matter effect of the earth in three-flavor oscillations. In the future the combination of long-base accelerator/reactor experiments and PINGU which detects the matter effect of atmospheric neutrinos is very likely to confirm the sign of Δm^2_{31} at 3σ confidence level.

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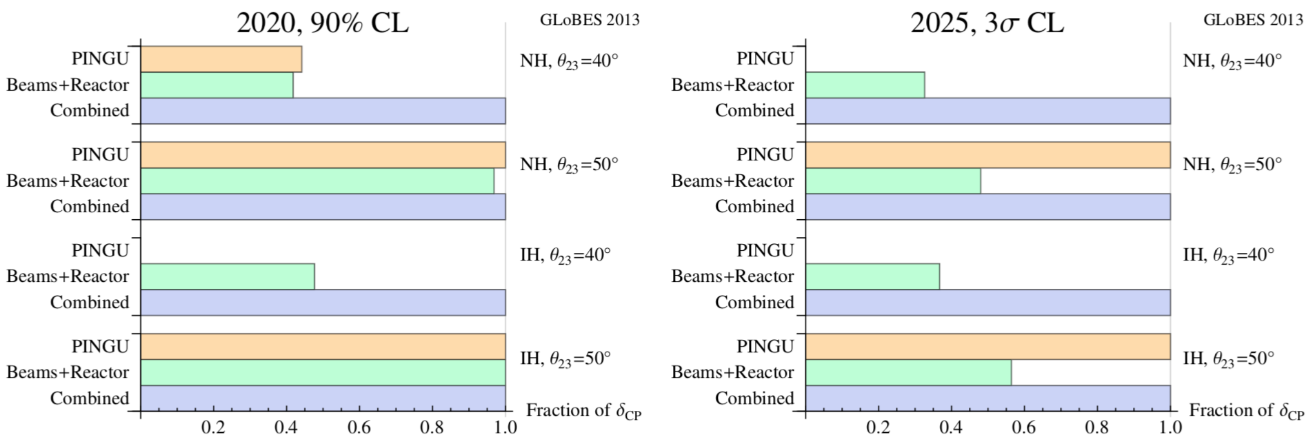

Figure 5. The fraction of CP phase, $\delta/2\pi$, for which the mass hierarchy can be determined by 3-year PINGU exposure (left figure) with 90% confident level and 8-year exposure at 3σ confident level. Performances for accelerator/reactor experiments and the combination of all these are also shown.

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