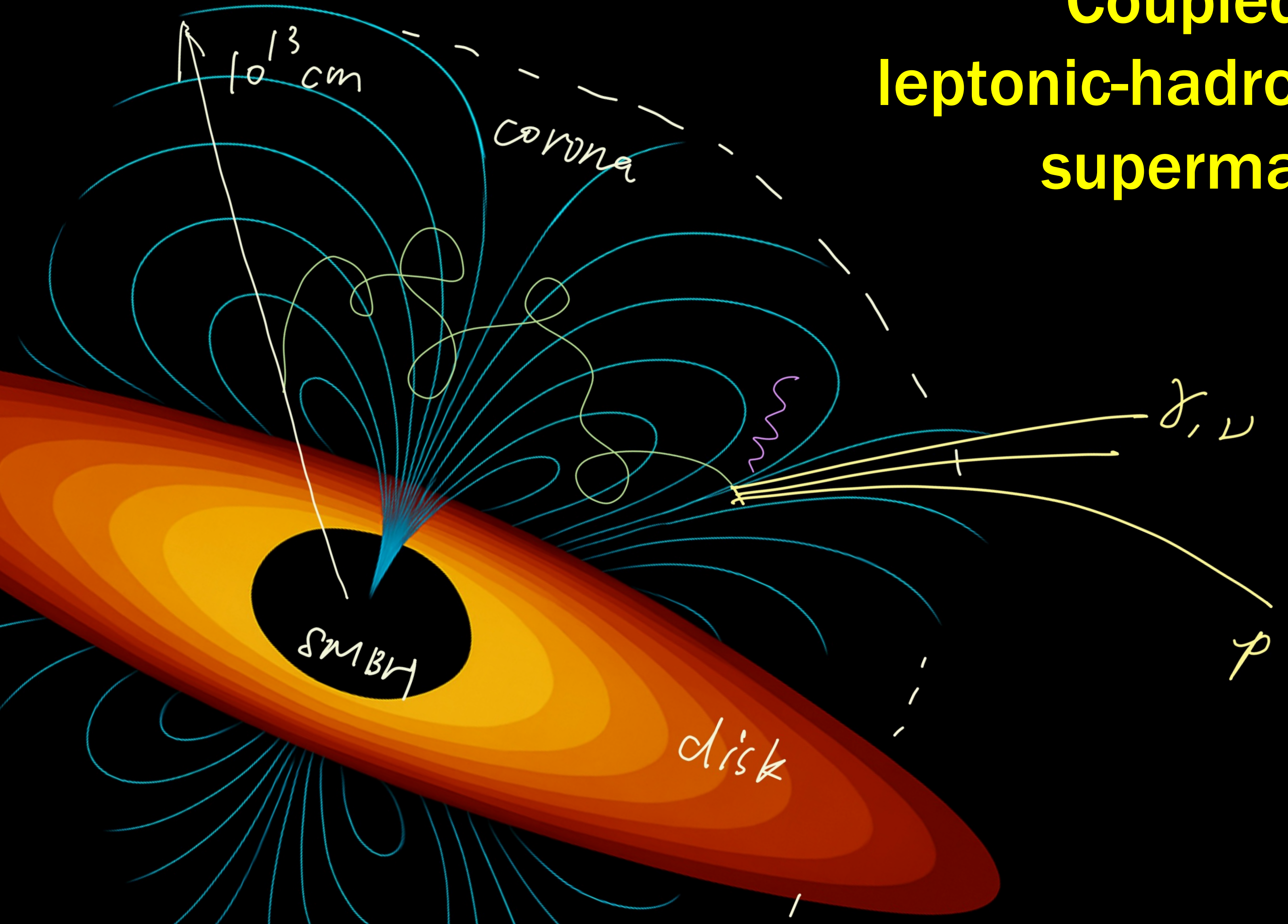


Coupled proton acceleration and leptonic-hadronic radiation in turbulent supermassive black hole coronae

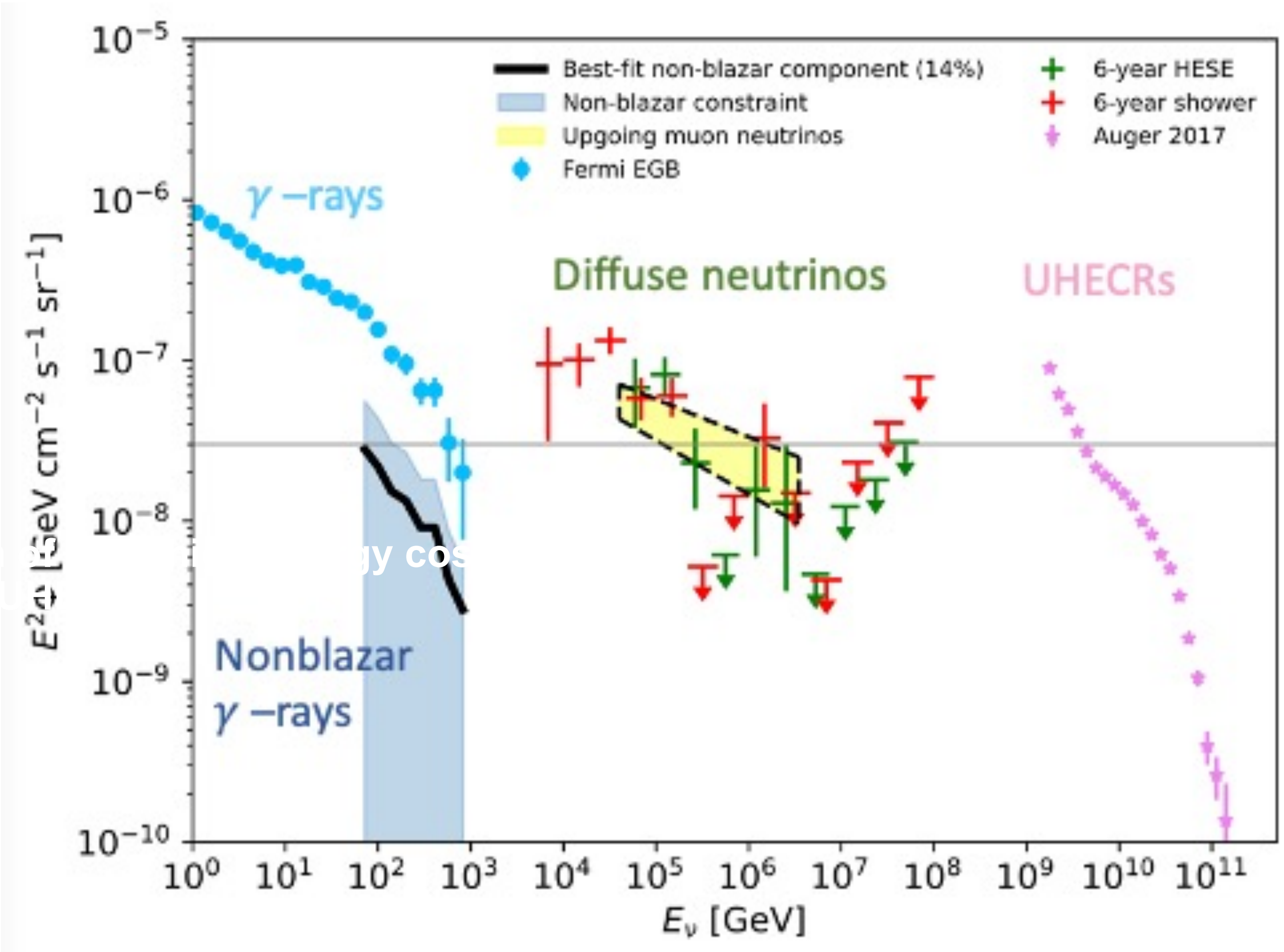
(arXiv: 2508.08233)

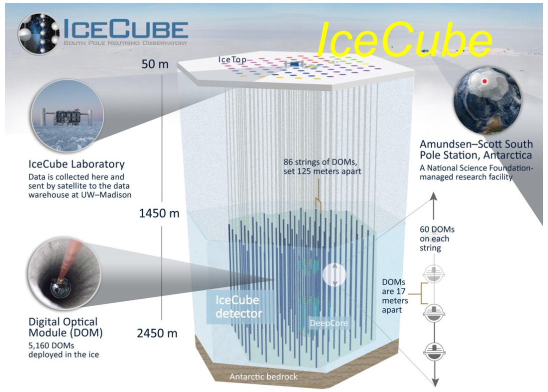
Chengchao Yuan
DESY

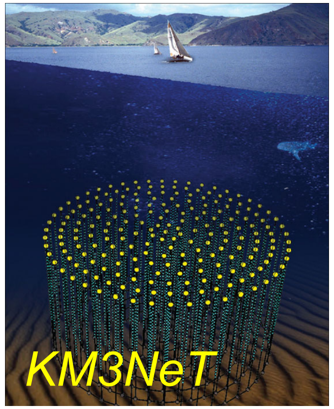
ULB, Brussels
2025/09/24

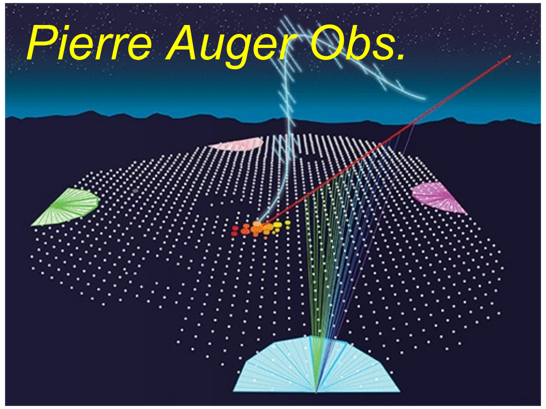
Contents

Cosmic rays, HE neutrinos, HE photons







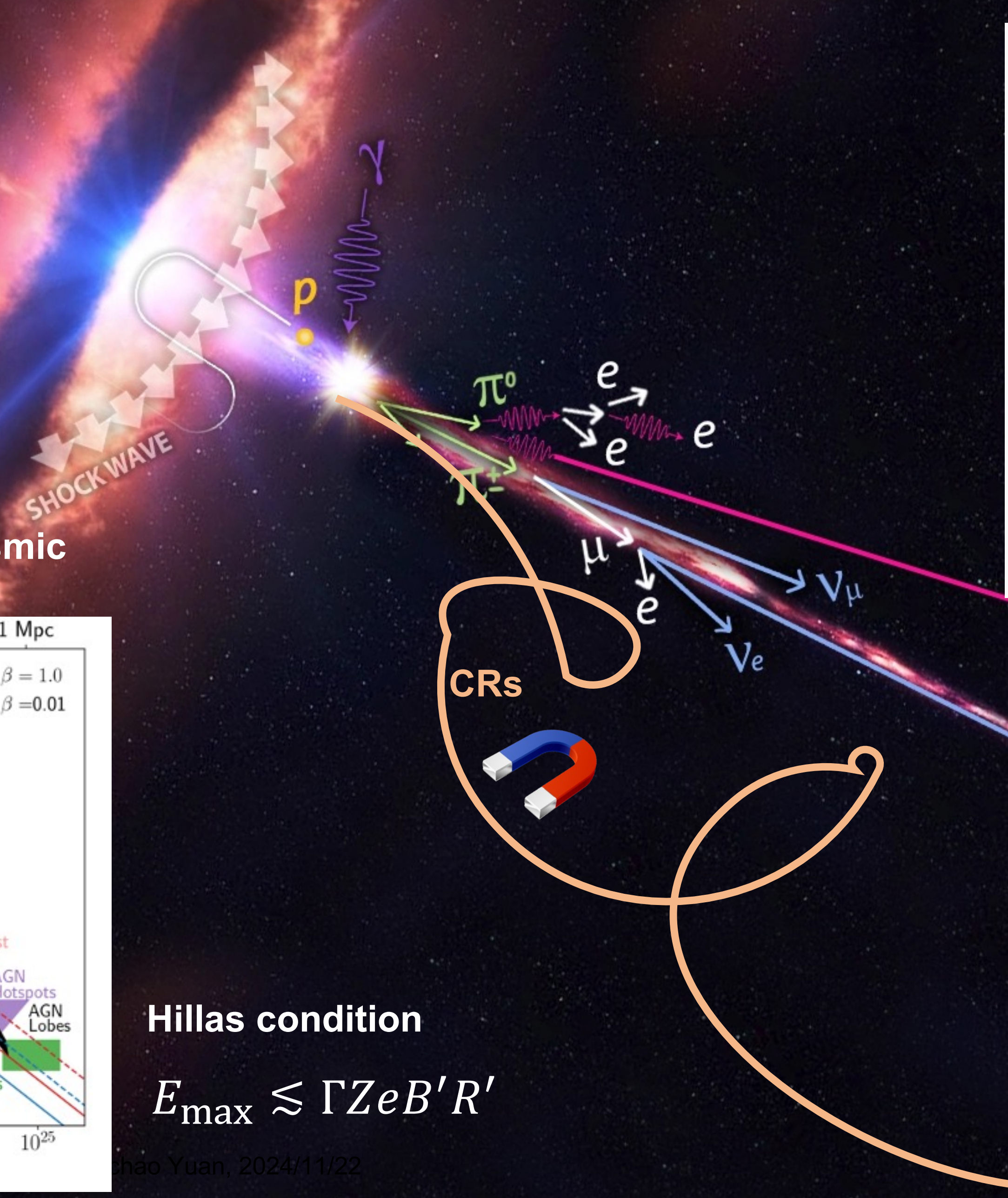


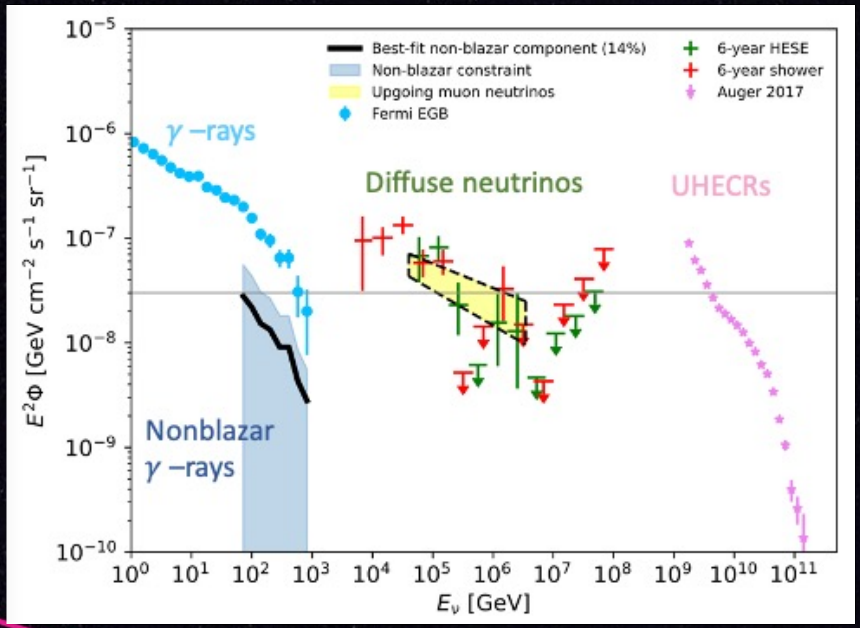
Gamma-rays: Fermi LAT, Imaging Atmospheric Cherenkov Telescopes (IACTs, e.g., CTA), …

HE Neutrinos: IceCube, KM3NeT, …

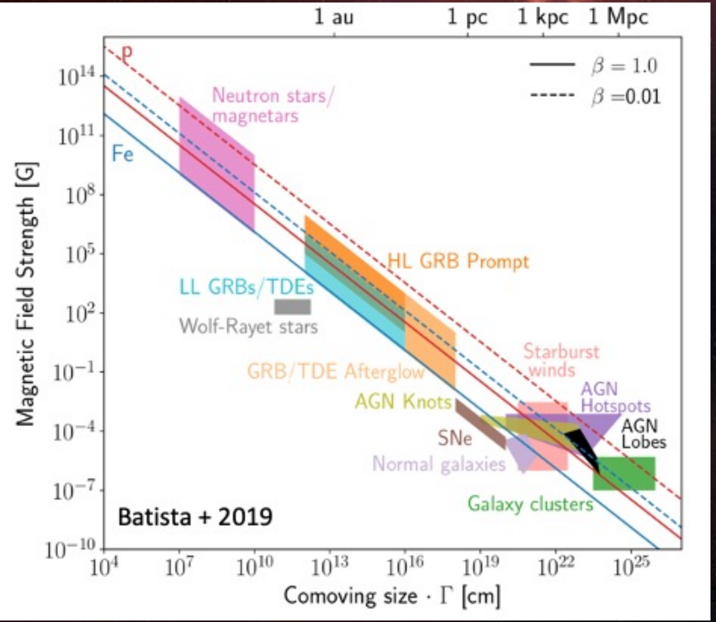
UHECRs: Pierre Auger Obs., Telescope Array (TA), Large High Altitude Air Shower Obs. (LHAASO)…

Cosmic rays, neutrinos, and electromagnetic (EM) signals from extreme sources





Origin of ultra-high-energy cosmic rays (UHECRs)



Hillas condition

$$E_{\max} \lesssim \Gamma Z e B' R'$$

(Image credit: IceCube/NASA)

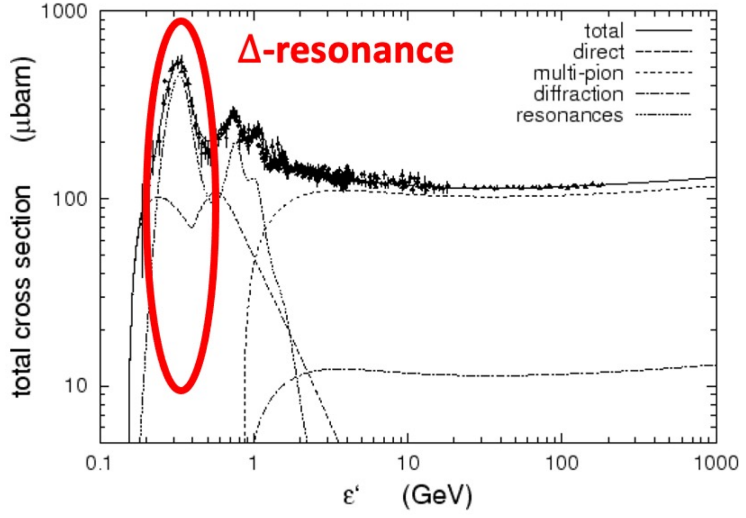
Production of High-Energy Astrophysical Neutrinos

Photo-pion/meson ($p\gamma$) process

$$p + \gamma \rightarrow \Delta^+ \rightarrow \pi^\pm/\pi^0 + X$$

Ingredients: dense (low-energy) target photons [e.g., ambient thermal photons] + CRs

Delta resonance proton energy: $E_p \gtrsim \frac{m_\pi(2m_p + m_\pi)c^2}{4\varepsilon_\gamma}$



$$\pi^\pm \rightarrow \mu^\pm + \nu_\mu(\bar{\nu}_\mu),\ \mu^\pm \rightarrow e^\pm + \bar{\nu}_\mu(\nu_\mu) + \nu_e(\bar{\nu}_e)$$

$$\pi^0 \rightarrow \gamma + \gamma$$

$$\varepsilon_\nu Q_{\varepsilon_\nu} \approx \frac{3K}{4(K+1)} e^{-f_{pp,p\gamma}} (\varepsilon_p Q_{\varepsilon_p})\,|_{\varepsilon_p \sim 20\varepsilon_\nu}$$

$$\varepsilon_\gamma Q_{\varepsilon_\gamma} \approx \frac{4}{3K} \varepsilon_\nu Q_{\varepsilon_\nu}\,|_{\varepsilon_\gamma \sim 2\varepsilon_\nu}$$

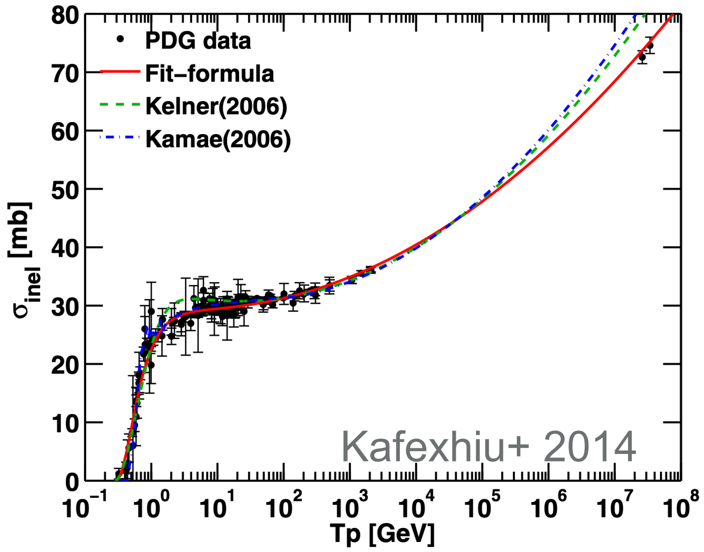
$$pp:\ K = N_{\pi^\pm}/N_{\pi^0} \sim 2$$

$$p\gamma:\ K = N_{\pi^\pm}/N_{\pi^0} \sim 1$$

Hadronuclear (pp) process

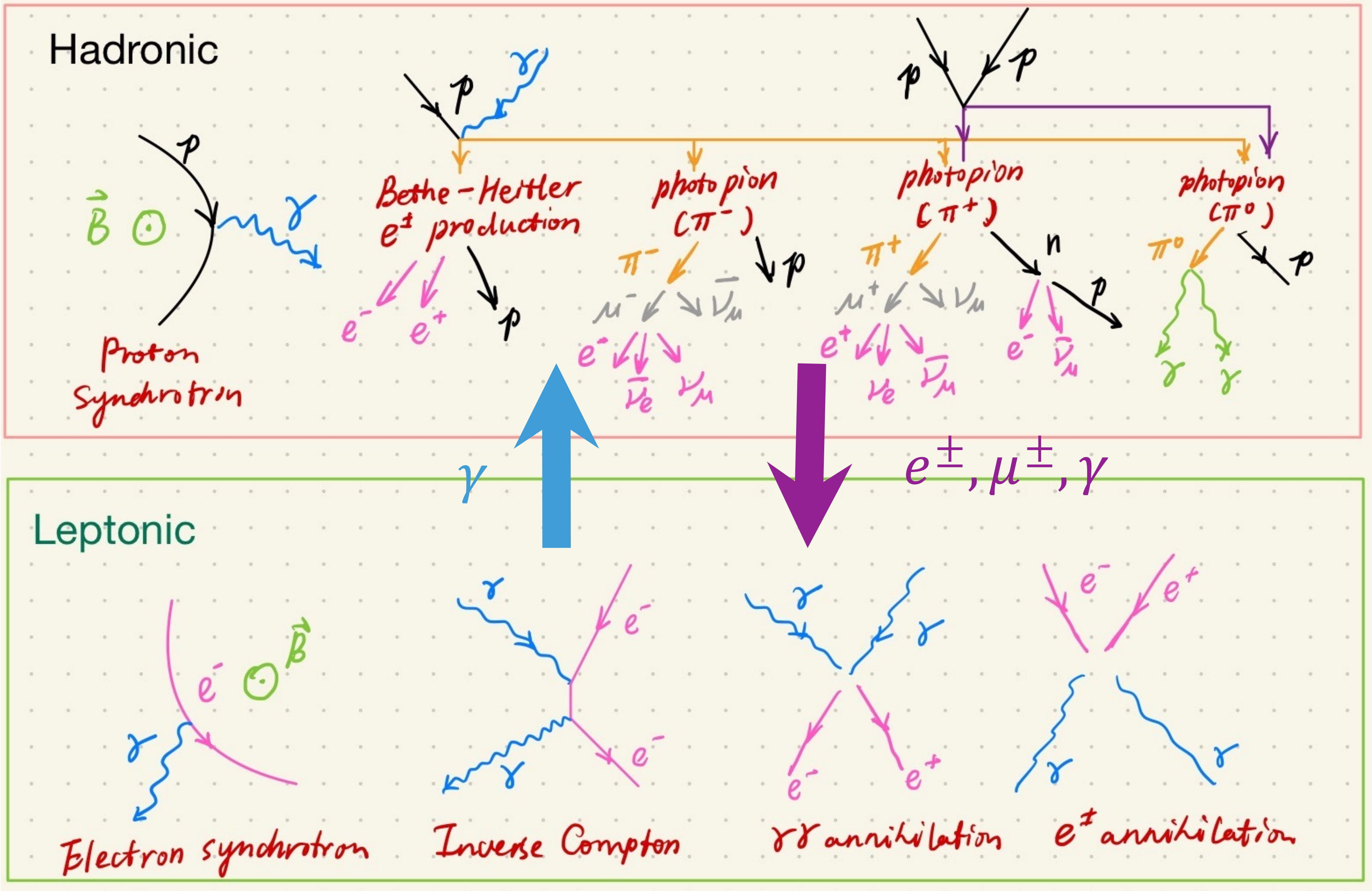
$$p + p \rightarrow \pi^+/\pi^0 + X$$

Ingredients: dense thermal/rest target + CRs



proton kinetic energy in the rest frame of the target proton

Neutrino production and electromagnetic (EM) cascades



A complete multi-messenger picture should include

- **Dynamics of the radiation zone (especially for transients/variable sources)**
- **Particle acceleration**
- **Radiation processes of accelerated particles**
- **Non-linear cascades from secondary particles (e.g., pions, muons, electrons)**

In practice, particle acceleration and radiation could occur in the same region.

Goal of this work:

To self-consistently couple time-dependent proton acceleration with the full radiation processes.

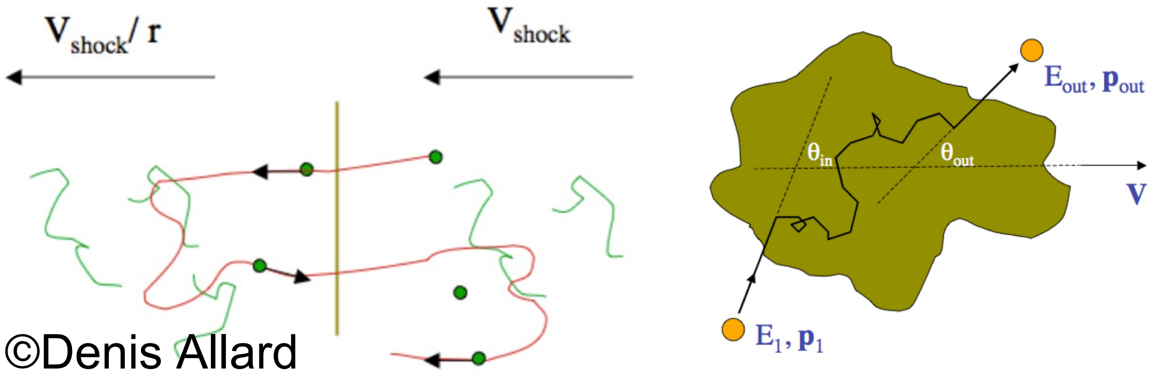
Particle acceleration mechanisms

- **Magnetization:**

$$\sigma_B \equiv \frac{u_B}{u_k} = \frac{\text{magnetic energy dens.}}{\text{plasma energy dens.}}$$

- **Spatial/temporal fluctuations:**
  Shocks, clouds, turbulences, reconnections

Fermi 1st-order (head-on) v.s. 2nd-order (random) acc.



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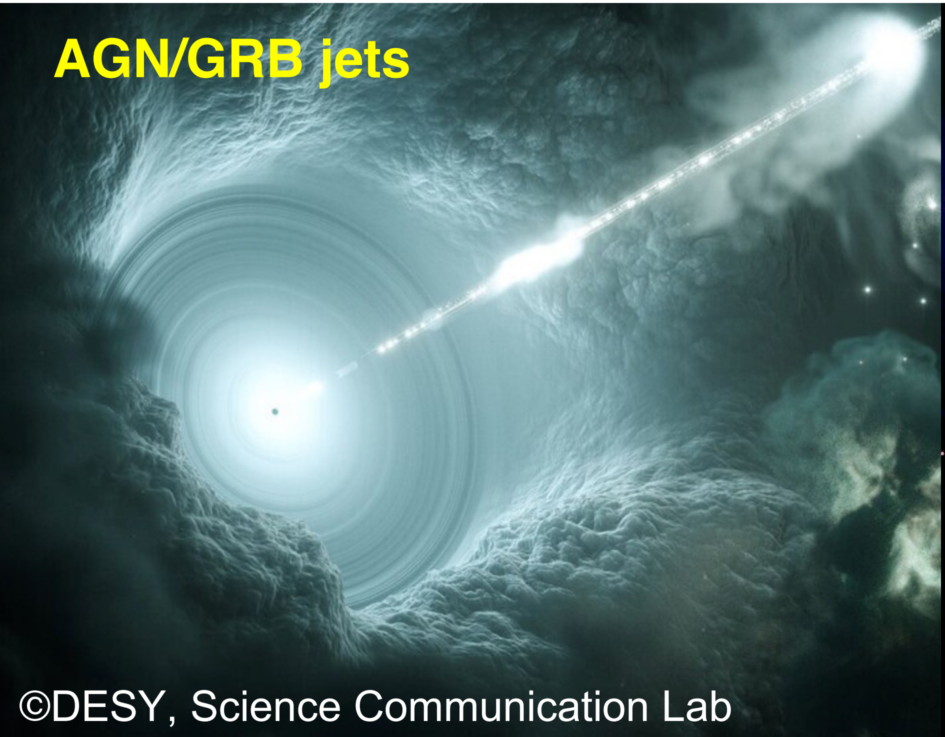

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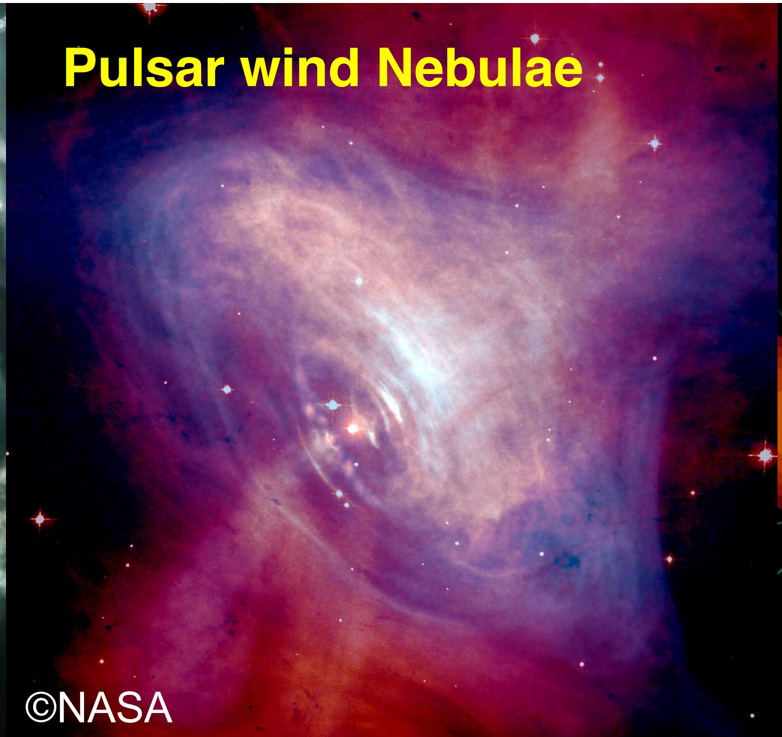

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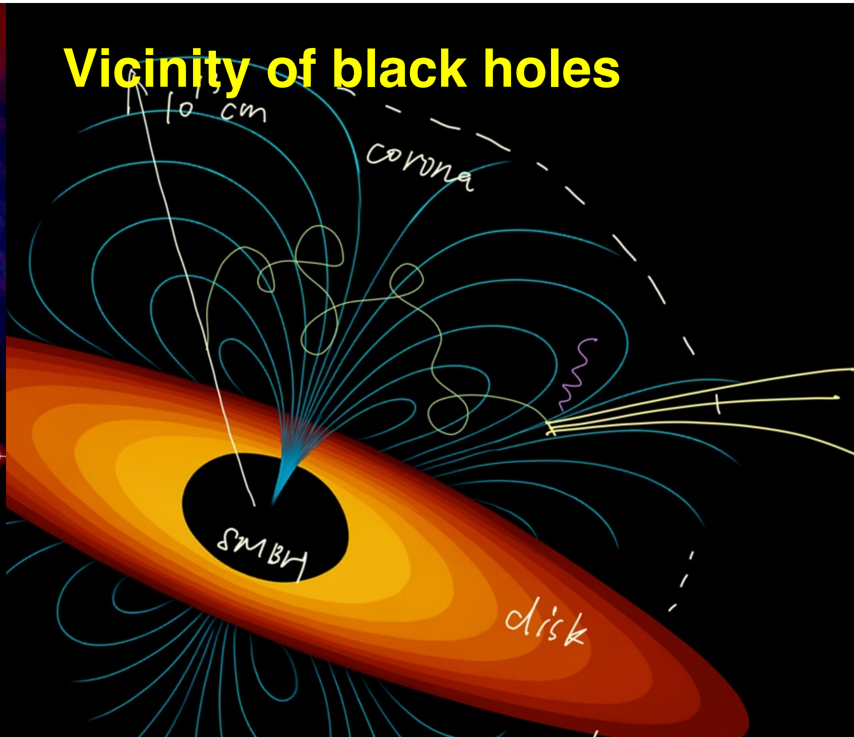
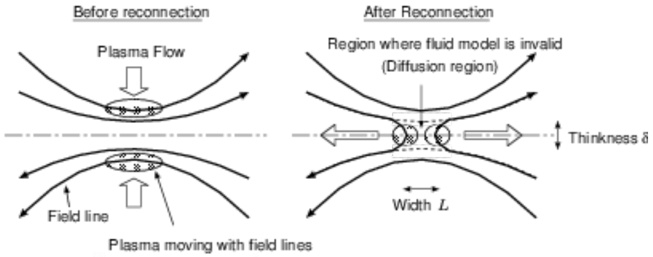


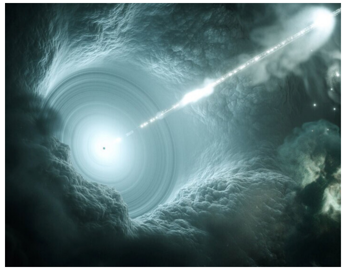
Particle acceleration mechanisms

Dependence on magnetization

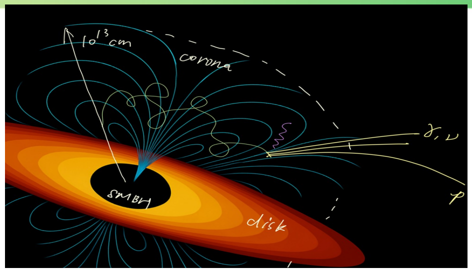
- Shocks (<10^-3): kinetic energy -> particles
- Magnetized turbulence (> 0.01): amplified magnetic field
- Magnetic reconnection (> 1): magnetic energy



(laminar) reconnection



turbulent acceleration



shock acceleration

weakly magnetized → *strongly magnetized /relativistic*

10^{-4} 0.01 1 σ

Fokker-Planck equation: proton distribution in *p*-space

$$\frac{\partial f}{\partial t} = \frac{1}{p^2}\frac{\partial}{\partial p}\left[p^2 D_p(p)\frac{\partial f}{\partial p} + \frac{p^3}{t_{\rm cool}} f\right] - \frac{f}{t_{\rm esc}} + q(p)$$

Diffusion in Momentum space | Energy loss from radiation cooling | Escape | Injection

$$\text{acceleration time}: t_{\rm acc} = p^2/D_p$$

Solving the FP equation

- Particle distribution

$$f(p,t) = \frac{dN_p}{dp^3 dV}$$

- Normalization to proton power

$$L_p = -4\pi c V \int p^2 D_p(p) \frac{\partial f}{\partial p} dp.$$

- Retrieving the energy spectra

$$n_p(E,t) = E d^2 N_p/(dEdV) = 4\pi p^3 f(p,t)|_{E=pc}$$

- Correct momentum flow: Chang-Cooper weighting (Chang & Cooper, 1970)

$$\Phi^n_{i+1/2} = D_{i+1/2}\frac{f^n_{i+1} - f^n_i}{\Delta p_{i+1/2}} + \dot{p}_{i+1/2}[(1-\delta) f^n_{i+1} + \delta f^n_i],$$

$$\delta = \frac{1}{w} - \frac{1}{e^w - 1} \qquad w = \Delta p_{i+1/2}\dot{p}_{i+1/2}/D_{i+1/2}$$

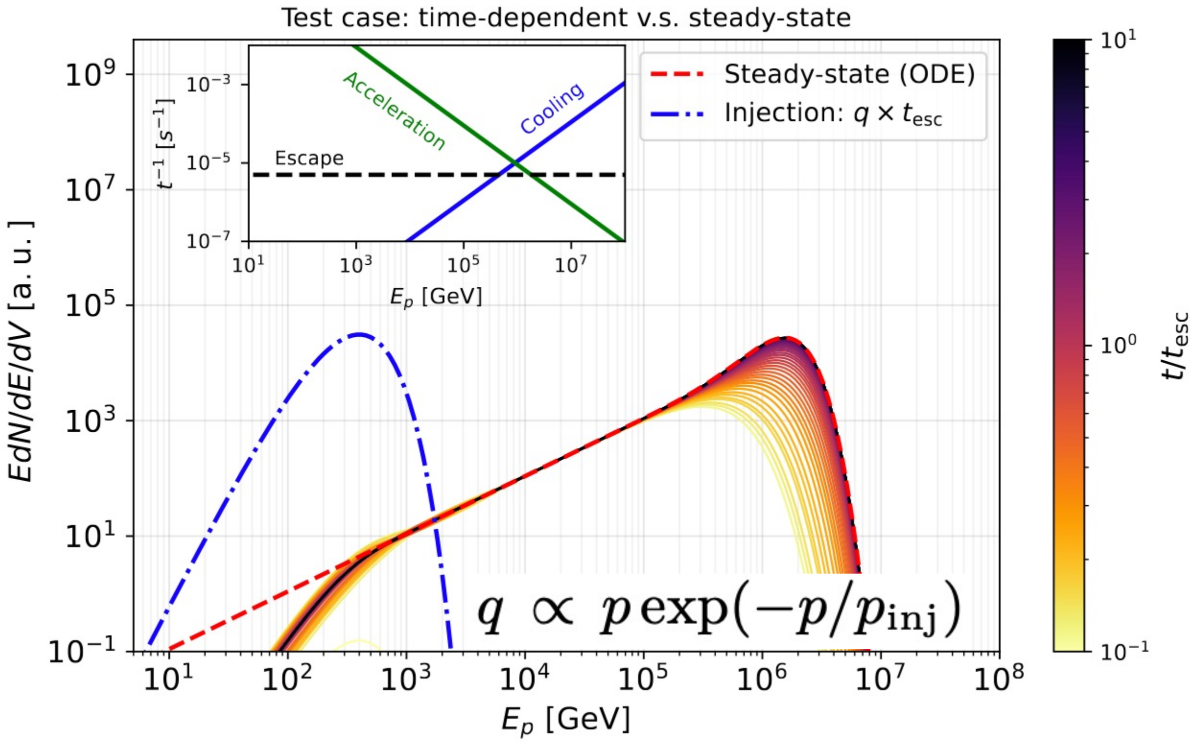
- Upwinding (delta->0) for exceedingly fast cooling (w >> 1)

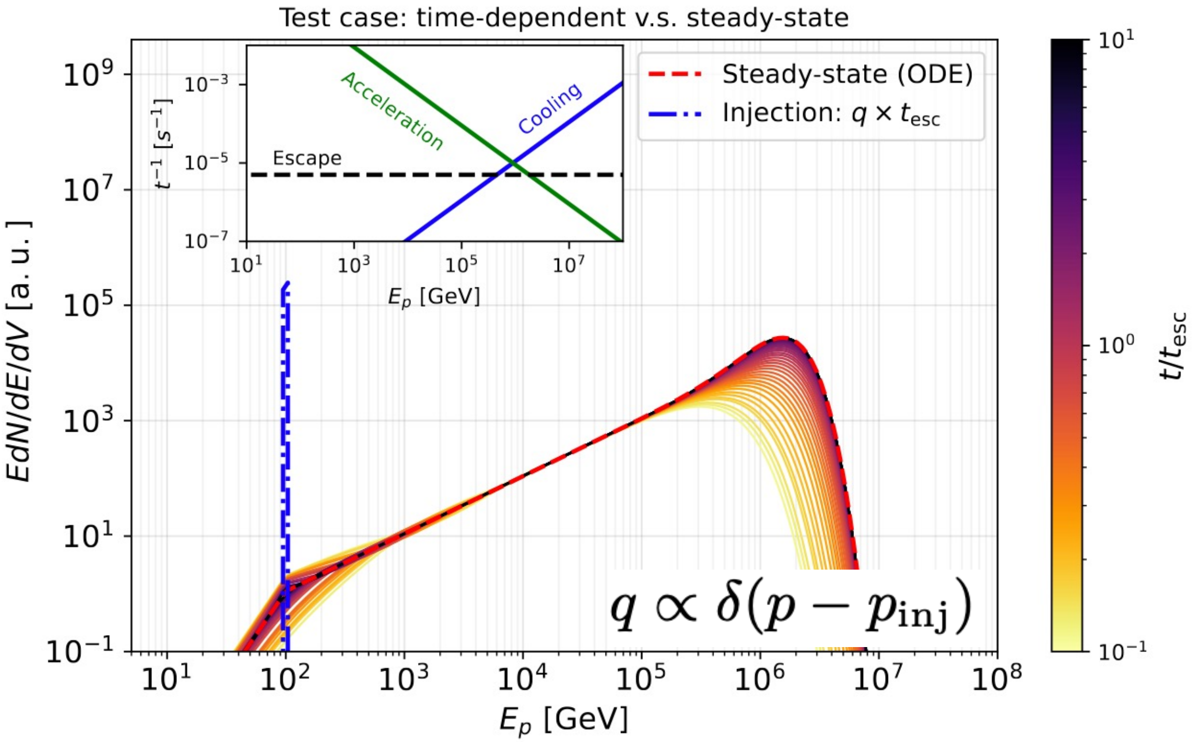
Fokker-Planck description: test cases

$t_{esc} = 2 \times 10^5$ s, $t_{acc} = 10^6 \left(\frac{p}{p_c}\right)$ s, $t_{cool} = 10^4 \left(\frac{p}{p_c}\right)^{-1}$ s, $p_c = 10^7 m_p c$

acceleration time : $t_{acc} = p^2 / D_p$

Steady state: setting $\partial f / \partial t = 0$, PDE -> ODE





Conclusions: (1)Time dependent solutions converges efficiently to steady states; (2) The peak energy is determined by $t_{acc}^{-1} = t_{cool}^{-1}$; (3) results are not sensitive to the injection function q

Yuan et al., 2025, arXiv: 2508.08233

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In practice, particle acceleration and radiation could occur in the same region.

Goal of this work:

To self-consistently couple time-dependent proton acceleration with the full radiation processes.

Radiation + cascades: AM3 software

AM3 = Astrophysical Multi-Messenger Modeling
Numerically solving the coupled PDEs for electron, proton, neutron, neutrino and photon distributions.

$$\partial_t n_i = \underbrace{Q_{i,ext}}_{\text{Injection}} + \sum_k Q_{int,k\rightarrow i} - \underbrace{\partial_E(\dot{E} \cdot n_i)}_{\text{Cooling}} - \underbrace{(\alpha_{i,esc} + \alpha_{i,adv})n_i}_{\text{Escape/Advection}}$$

- An ***Open-Source*** Tool for ***Time-Dependent*** Lepto-Hadronic Modeling of Astrophysical Sources
- Blazars, GRBs, TDEs, etc

(Klinger, Rudolph, Rodrigues, CY +, arXiv: 2312.13371, ApJS)

- Dedicated for **single-zone** isotropic multi-messenger (EM and neutrinos) emissions
- Efficient with optimized solvers
- Reliable as it reproduces the results from other codes, e.g., Hadronic Code Comparison Project (Cerruti et al. 2021)
- Well documented and easy to use

Code is now public! https://am3.readthedocs.io/en/latest/

So far, more than 15 papers (Blazars, GRBs, and TDEs) are published based on AM3.
More applications are expected in the near future!

Radiation + cascades: AM3 software

Numerically solving the coupled PDEs for electron, proton, neutrons, neutrino and photon distributions.

$$\partial_t n_i = \underbrace{Q_{i,ext} + \sum_k Q_{int,k\to i}}_{\text{Injection}} - \underbrace{\partial_E(\dot{E}\cdot n_i)}_{\text{Cooling}} - \underbrace{(\alpha_{i,esc} + \alpha_{i,adv})n_i}_{\text{Escape/Advection}}$$

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Solver optimization

- dimensionless $x = \ln(E/E_0)$
- cooling term $A = \dot{E}/E$,
- injection $\epsilon = EQ(E)$
- escape α
- Tridiagonal matrix solver + analytical solver

Electrons/positrons

$$\partial_t N_e = -\partial_x[A_e \cdot N_e - B_e \cdot \partial_x N_e] - (\alpha_{e,esc} + \alpha_{e,annihi})N_e + \epsilon_{e,ext} + \sum \epsilon_{e,internal}$$

Neutrinos

$$\partial_t N_\nu = -\alpha_{\nu,esc} N_\nu + \sum \epsilon_{\nu,int}$$

Intermediate particles: $\mu^\pm, \pi^\pm, \pi^0$

Photons

$$\partial_t N_\gamma = -(\alpha_{\gamma,esc} + \alpha_{\gamma,ssc} + \alpha_{\gamma,ic} + \alpha_{\gamma,\gamma\gamma} + \alpha_{\gamma,BH} + \alpha_{\gamma,p\gamma})N_\gamma + \epsilon_{\gamma,ext} + \sum \epsilon_{\gamma,intern}$$

Protons

$$\partial_t N_p = -\partial_x[A_p \cdot N_p - B_p \cdot \partial_x N_p] - (\alpha_{p,esc} + \alpha_{p,p\gamma} + \alpha_{p,pp})N_p + \epsilon_{p,ext}$$

Radiation + cascades: AM3 software

Numerically solving the coupled PDEs for electron, proton, neutrons, neutrino and photon distributions.

$$\partial_t n_i = Q_{i,ext} + \sum_k Q_{int,k\to i} - \partial_E(\dot{E}\cdot n_i) - (\alpha_{i,esc} + \alpha_{i,adv})n_i$$

Injection   Cooling   Escape/Advection

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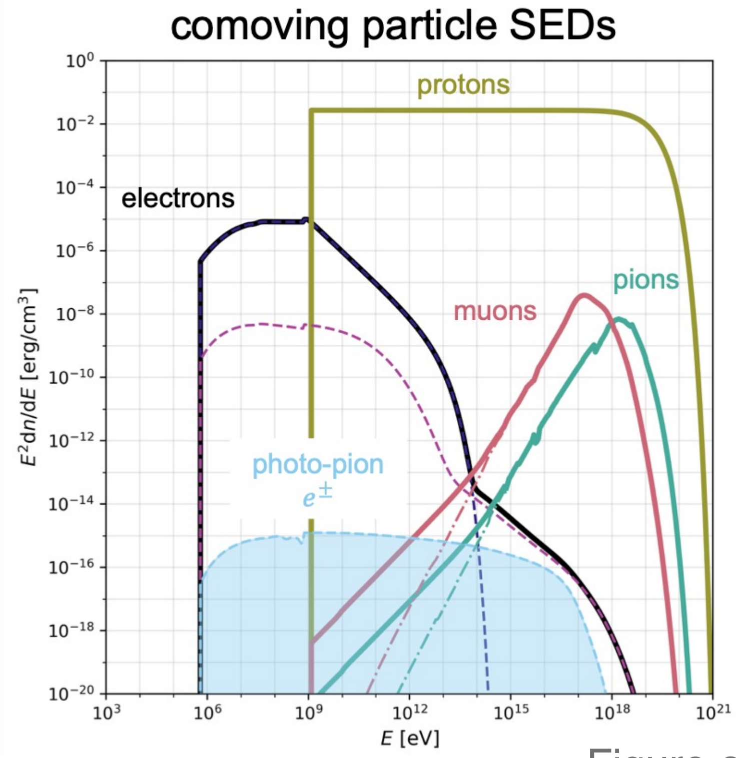
(Klinger, Rudolph, Rodrigues, CY +, arXiv: 2312.13371, ApJS)

Trackable photo-pion cascade:

$p\gamma \to \pi \to \mu \to e/\gamma$

- Injected protons
- pions
- muons
- primary electrons, secondary electrons
- Photon components



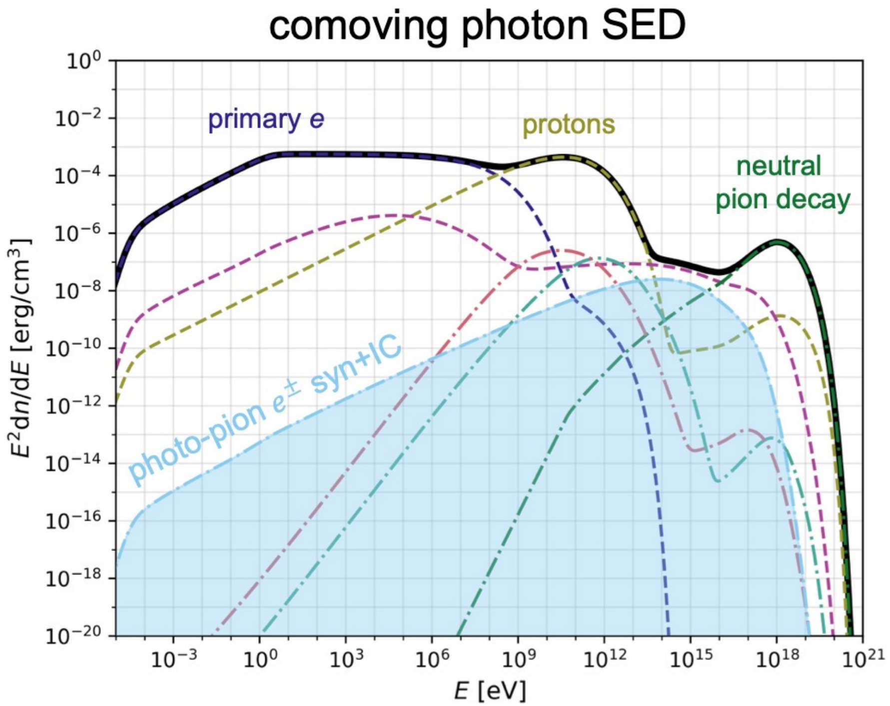


Figure credit: Marc Klinger

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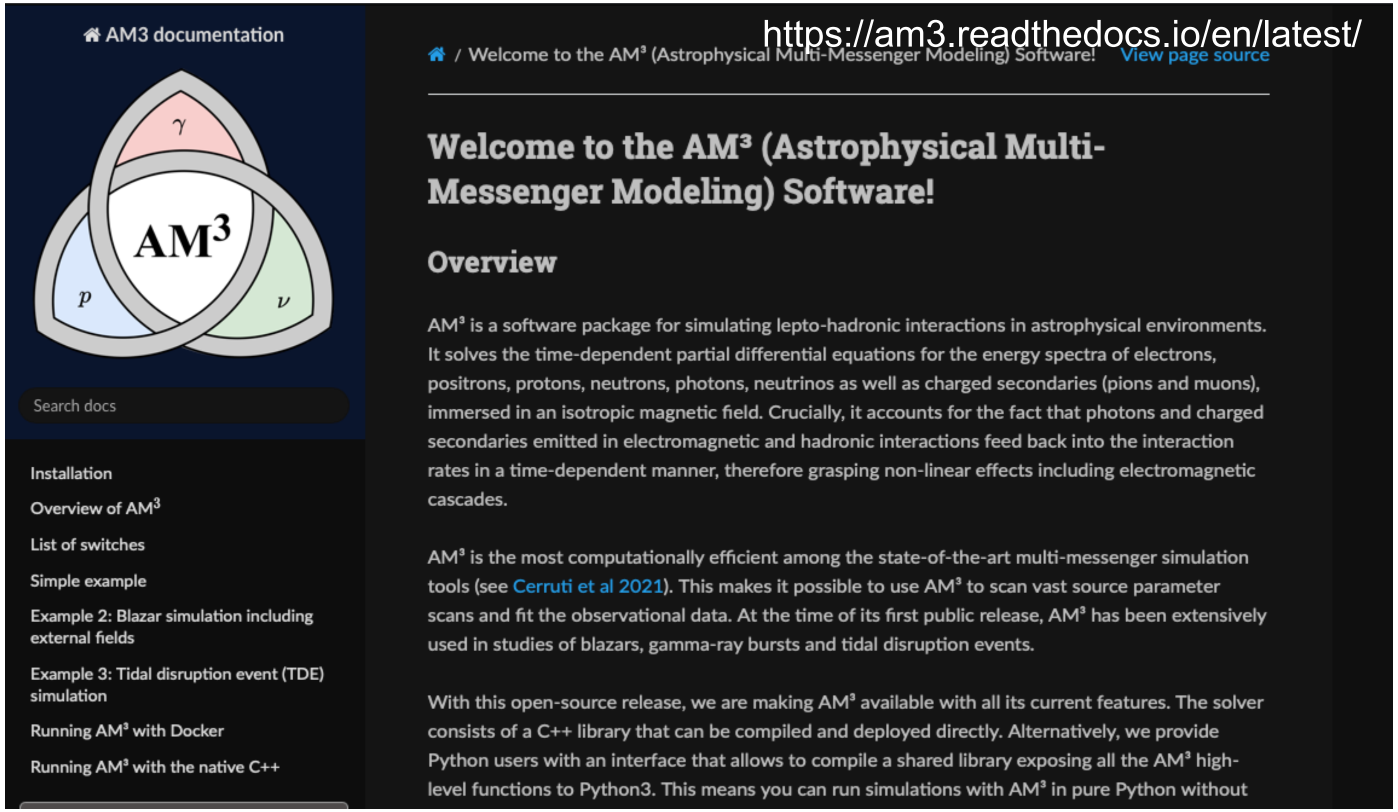
https://am3.readthedocs.io/en/latest/


AM3 documentation

Search docs

Installation
Overview of AM³
List of switches
Simple example
Example 2: Blazar simulation including external fields
Example 3: Tidal disruption event (TDE) simulation
Running AM³ with Docker
Running AM³ with the native C++

/ Welcome to the AM³ (Astrophysical Multi-Messenger Modeling) Software! View page source

Welcome to the AM³ (Astrophysical Multi-Messenger Modeling) Software!

Overview

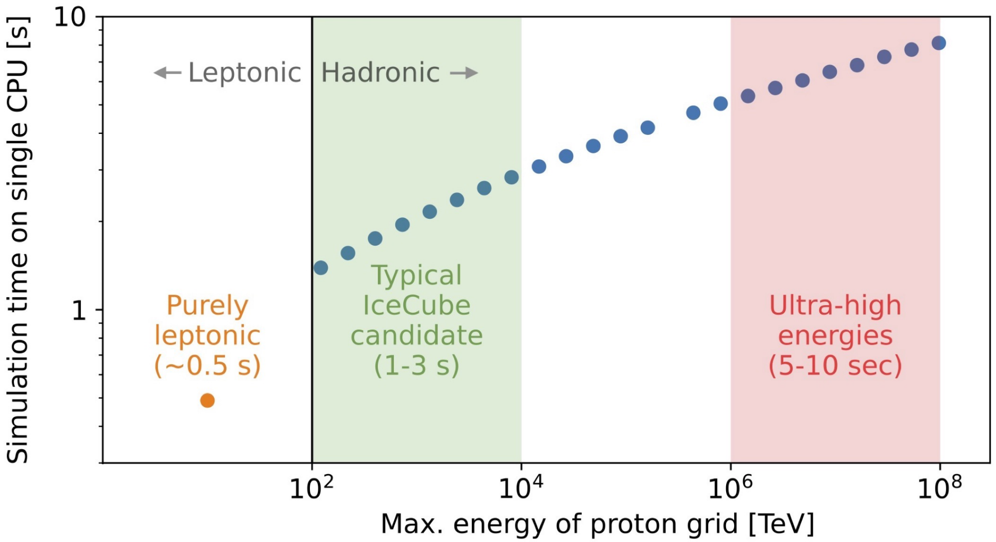
AM³ is a software package for simulating lepto-hadronic interactions in astrophysical environments. It solves the time-dependent partial differential equations for the energy spectra of electrons, positrons, protons, neutrons, photons, neutrinos as well as charged secondaries (pions and muons), immersed in an isotropic magnetic field. Crucially, it accounts for the fact that photons and charged secondaries emitted in electromagnetic and hadronic interactions feed back into the interaction rates in a time-dependent manner, therefore grasping non-linear effects including electromagnetic cascades.

AM³ is the most computationally efficient among the state-of-the-art multi-messenger simulation tools (see Cerruti et al 2021). This makes it possible to use AM³ to scan vast source parameter scans and fit the observational data. At the time of its first public release, AM³ has been extensively used in studies of blazars, gamma-ray bursts and tidal disruption events.

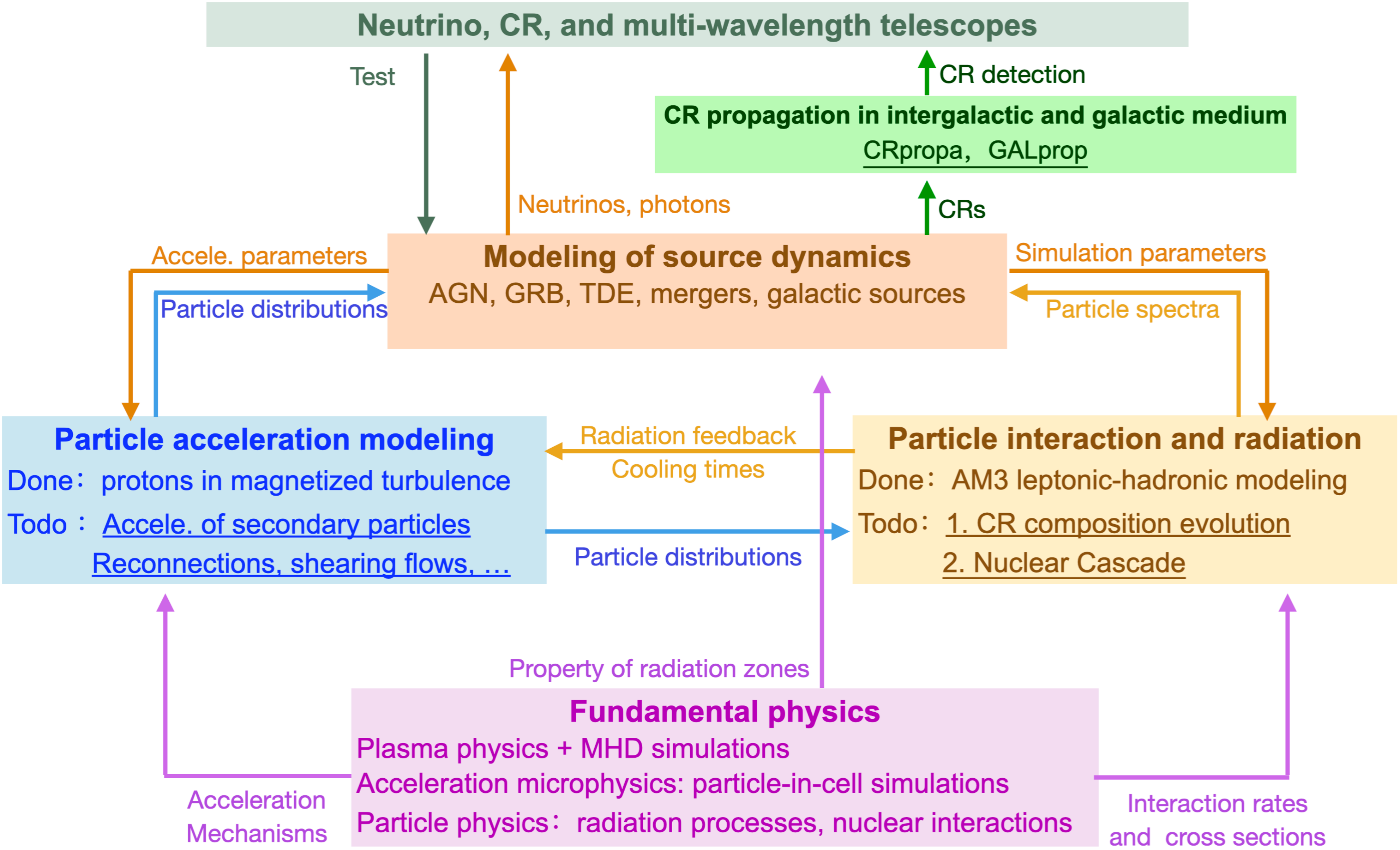
With this open-source release, we are making AM³ available with all its current features. The solver consists of a C++ library that can be compiled and deployed directly. Alternatively, we provide Python users with an interface that allows to compile a shared library exposing all the AM³ high-level functions to Python3. This means you can run simulations with AM³ in pure Python without

Performance: C++ source code, python interface

- Simulation: kernel initialization, particle injection, ~30 steps to steady state
- Tested on a single CPU on Apple M2 chip
- Very fast for lepto-hadronic simulations (< 0.5 s/step)



Coupled proton acceleration with leptonic-hadronic radiation

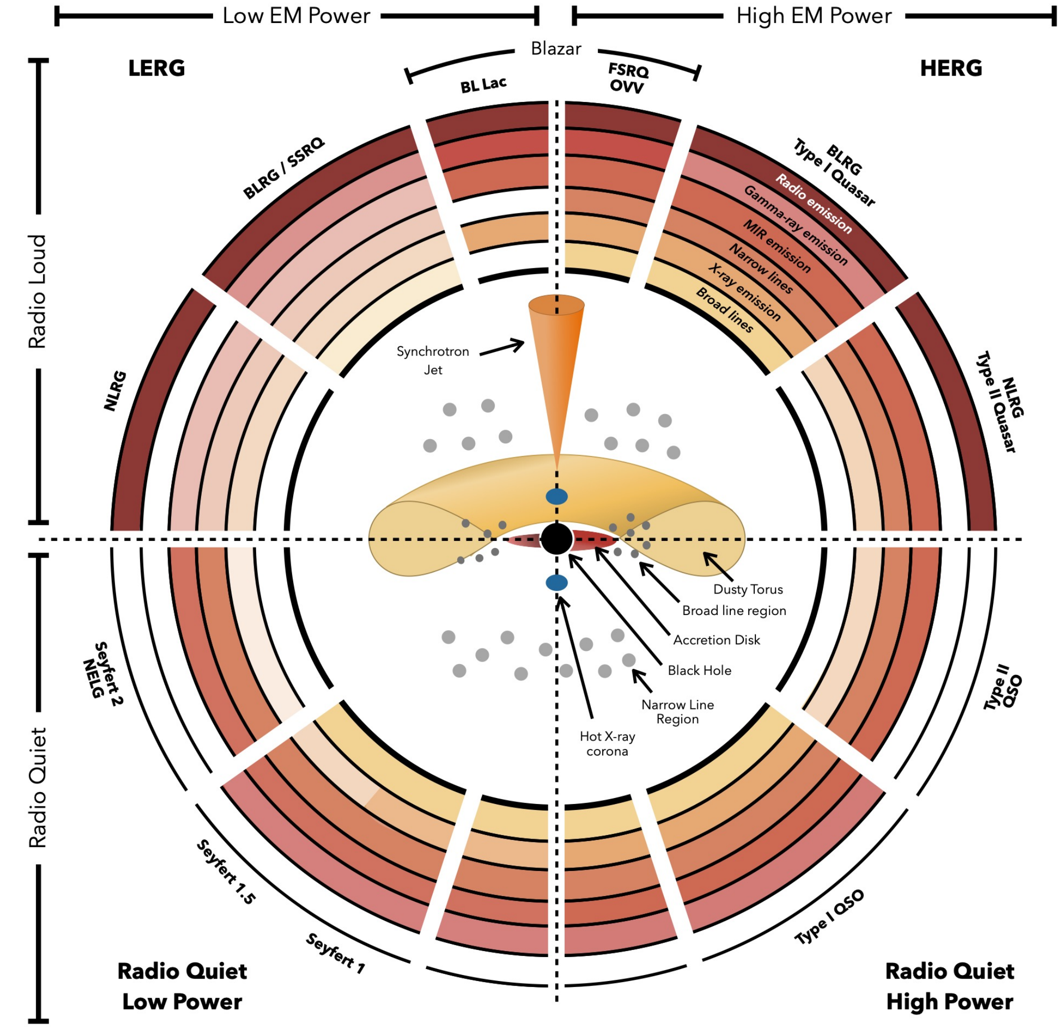


Part 2

Steady corona in Seyfert galaxy NGC 1068

NGC 1068

- One of the best studied active galactic nuclei (AGNs)
- Seyfert galaxy (a subclass of AGN, radio quiet but with bright core)
- SMBH mass ~ 10^7 solar mass
- Nearby AGN: luminosity distance ~ 10 Mpc
- Bright X-ray and infrared luminosity
- The 1st steady neutrino source identified by IceCube

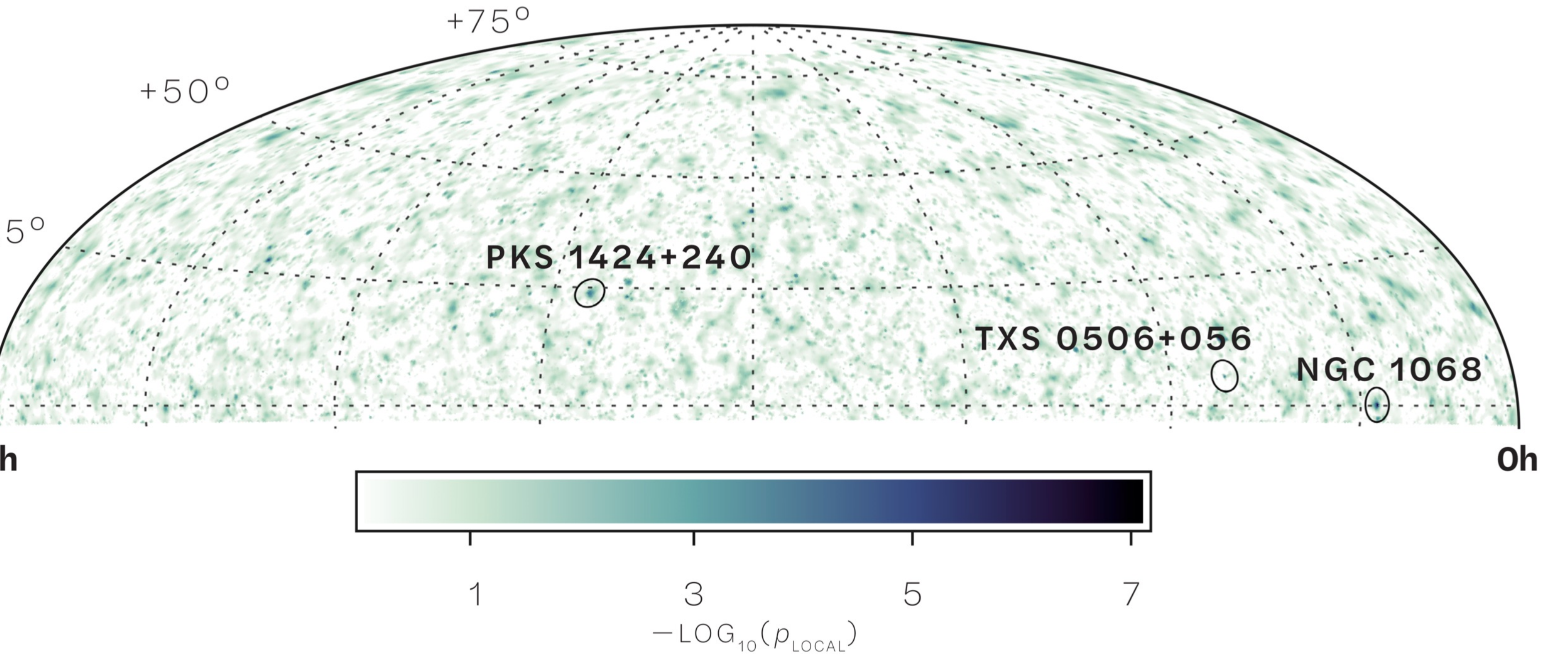


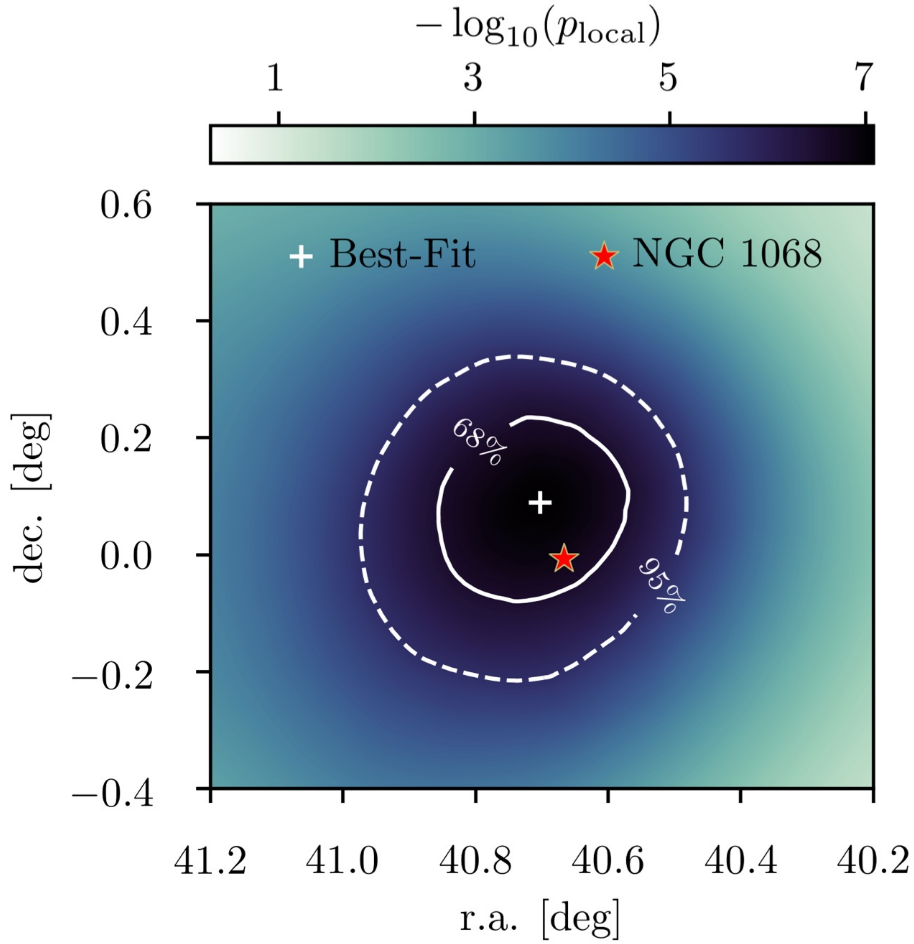
J. E. Thorne

NGC 1068: neutrino association

Skymap of the scan for point sources in the Northern Hemisphere.

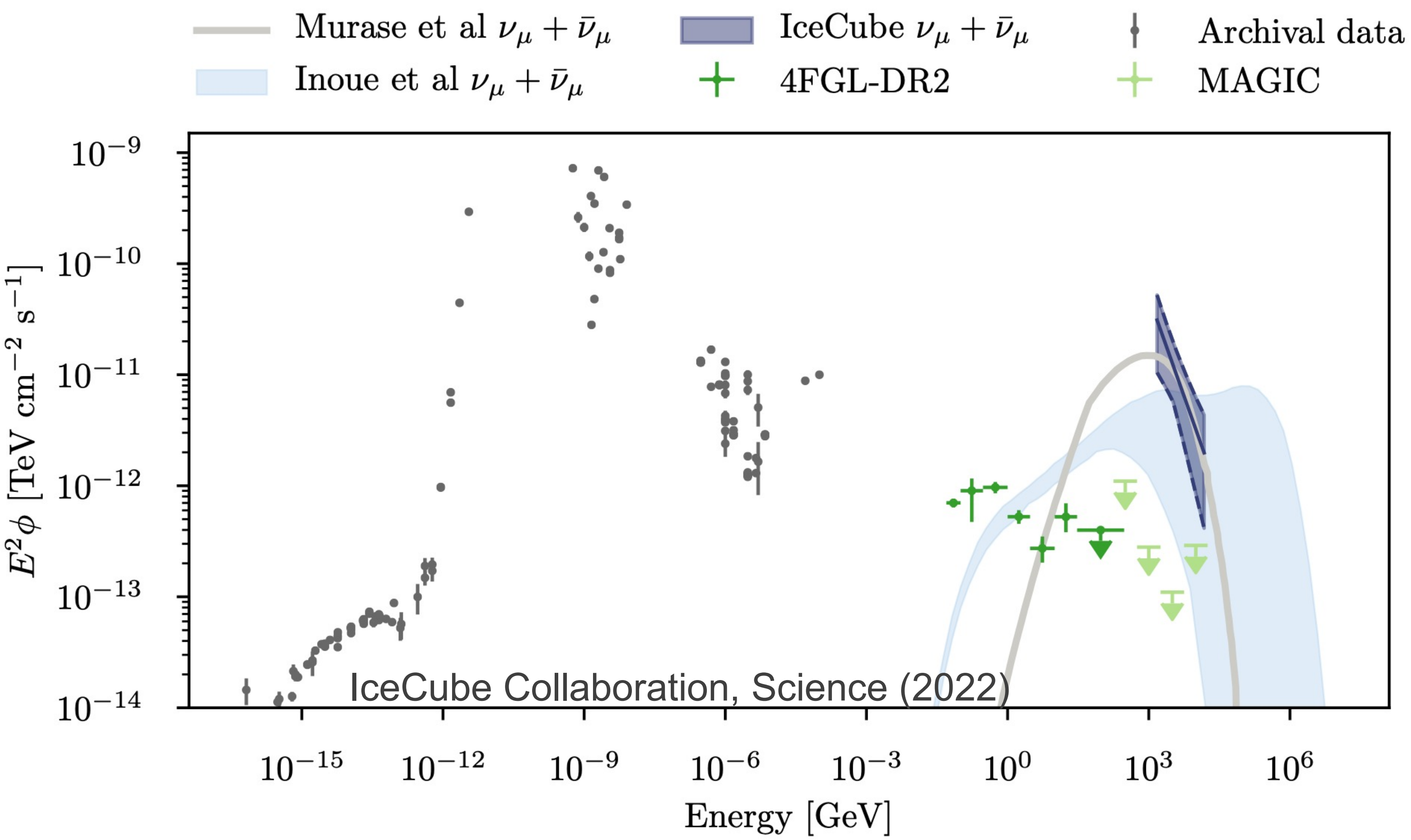
IceCube Collaboration, Science (2022)





An excess of ~ 79 neutrinos associated with NGC 1068 (significance: 4.2 sigma)

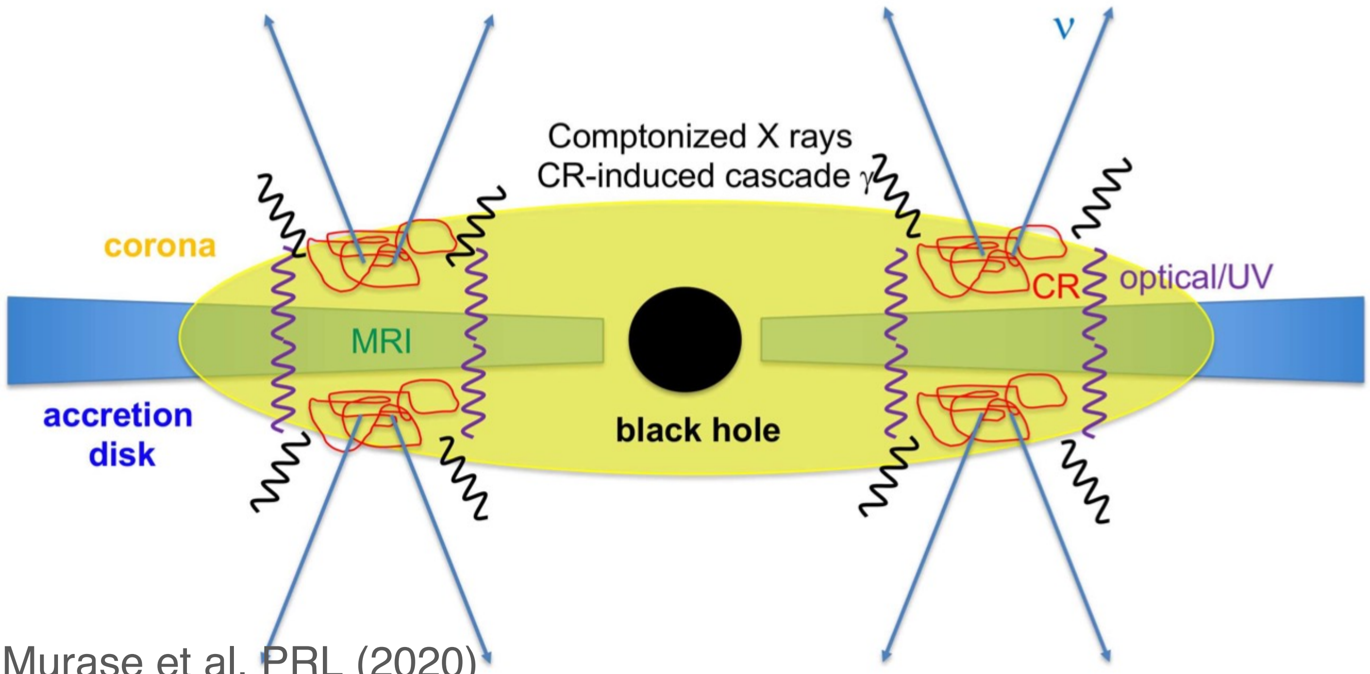
NGC 1068: spectal energy distribution (SED)



- Neutrino spectra: a single power-law distribution from ~10 TeV to PeV energies
- Neutrino flux is much higher than the gamma-ray flux by Fermi LAT
- Gamma rays are strongly ***obscured***
- Neutrinos are generated in the dense core region (<100 Rg)

NGC 1068: the coronal scenario

Corona: a compact, hot, magnetized electron plasma sitting very close to the SMBH and inner accretion disk



Murase et al, PRL (2020)

See also: Inoue et al. 2020; Kheirandish et al. 2021; Padovani et al. 2024; Fiorillo et al. 2024; Mbarek et al. 2024; Saurenhaus et al. 2025

The physical picture:

- Compact region R ~ 10-100 Rg
- Electron temperature ~ 100 keV
- Comptonized

$$\tau_T = n_e \sigma_T R \sim 0.1 - 1$$

- Protons could be accelerated by turbulences
- Interactions with thermal protons and disk/coronal X-rays produce neutrinos
- $\gamma\gamma$ annihilation suppresses gamma-ray flux

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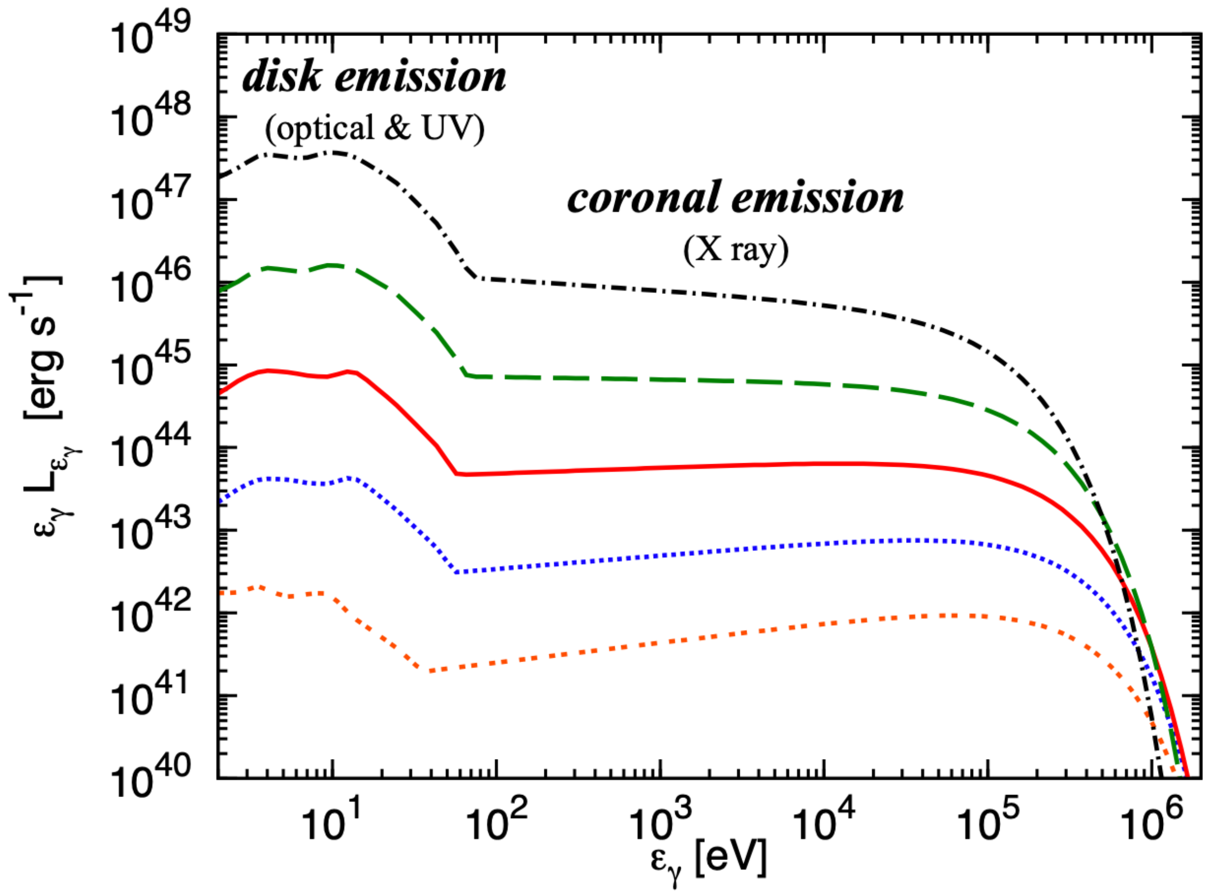

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NGC 1068: proton acceleration in turbulent corona

Parameters:

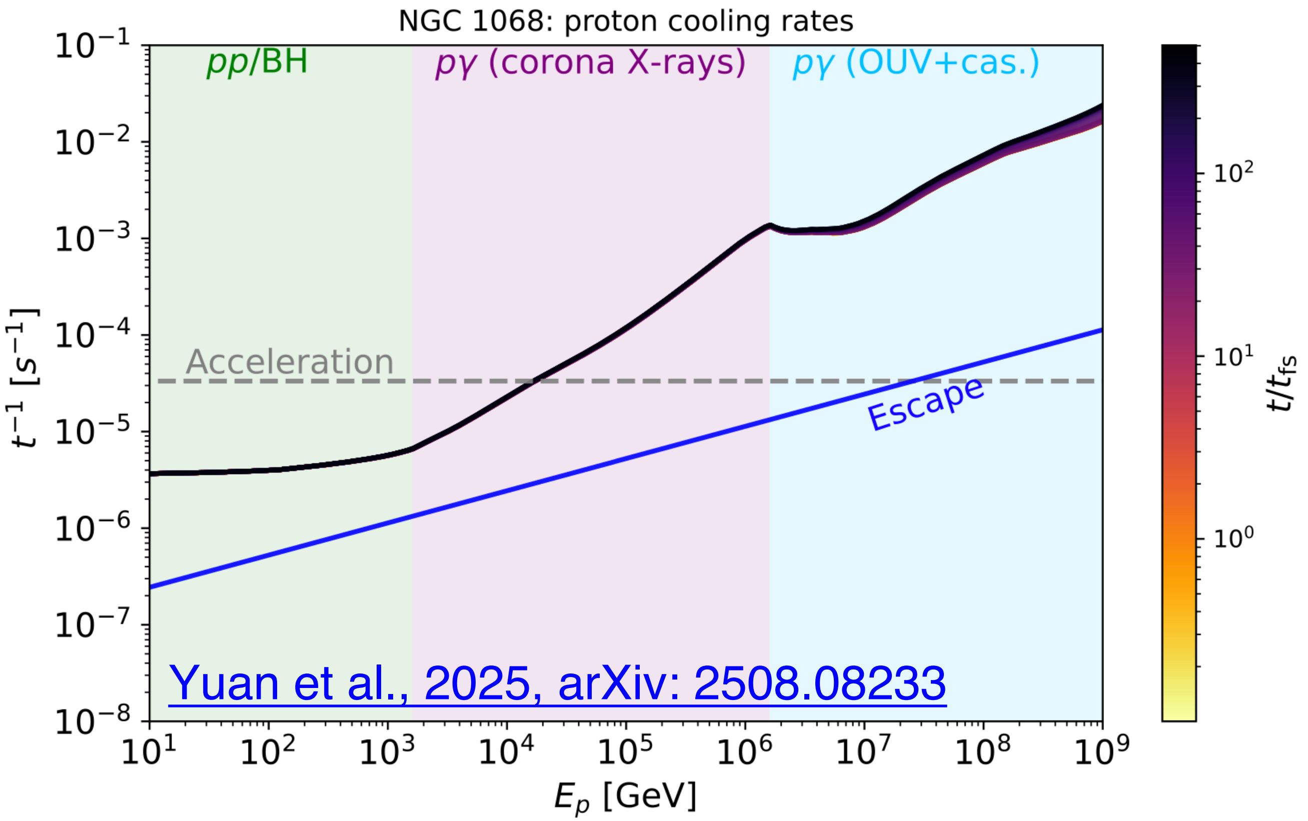
- Corona radius 20 Rg
- Magnetization: 0.1
- Thomson opacity: 0.5 (Ricci et al., 2018)
- Electron temperature ~ 100 keV
- Proton accelerate timescale (inferred from PIC simulations, Comisso & Sironi, 2019)

$$t_{\rm acc} \equiv \frac{p^2}{D_p} = \frac{10\, l_{\rm cl}}{\sigma_{\rm tur} c} \simeq 10^4 M_7 \left(\frac{\mathcal{R}}{20}\right)\left(\frac{\eta_{\rm cl}}{\sigma_{\rm tur}}\right) \ {\rm s.}$$

- Escape timescale depend on the gyro-radius ($0.3 < \zeta < 0.5$, Lemoine, 2023; Kempski et al., 2023)

$$l_r = E_p/(eB) \quad \lambda_{\rm mfp} = l_{\rm cl}(l_r/l_{\rm cl})^{\zeta}$$

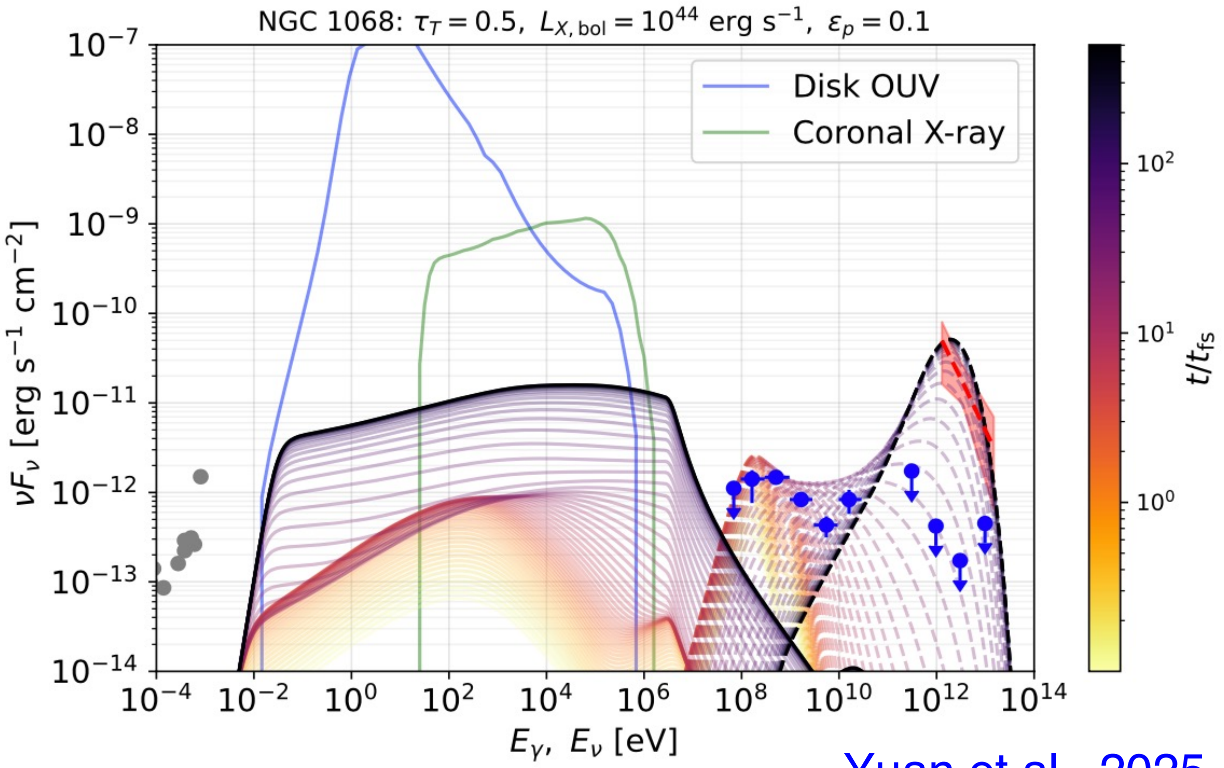
$$t_{\rm esc} = R_{\rm co}^2/(\lambda_{\rm mfp} c)$$

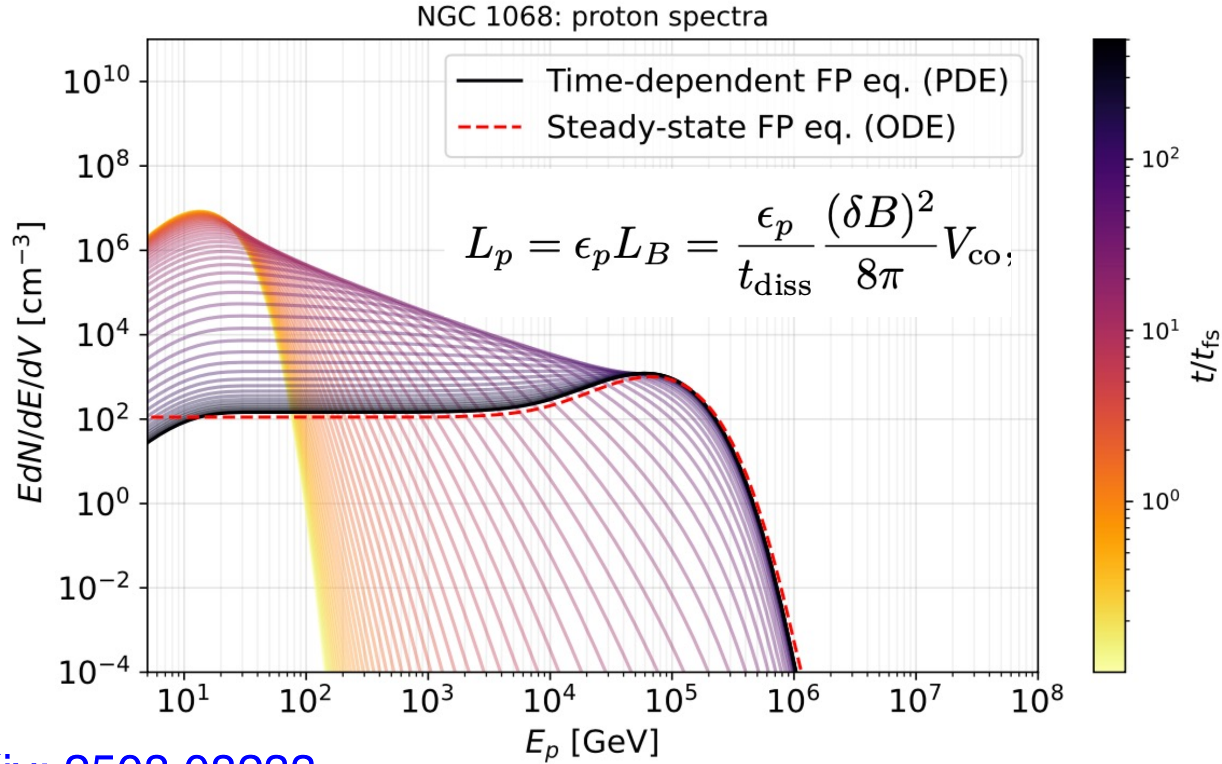


Proton cooling:

- pp/pγ interactions with disk/coronal X-rays and cascade emissions (radiation feedback)
- Bethe-Heitler pair production

NGC 1068: neutrino, photon, and proton spectra

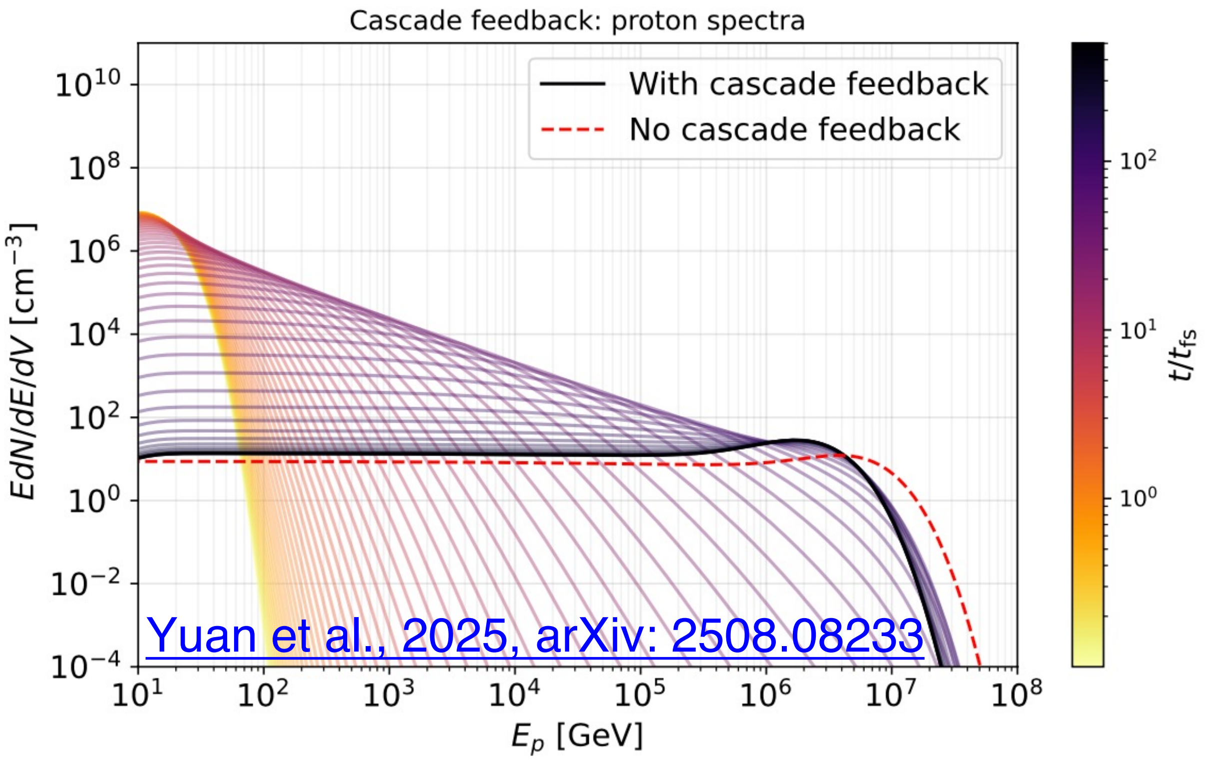


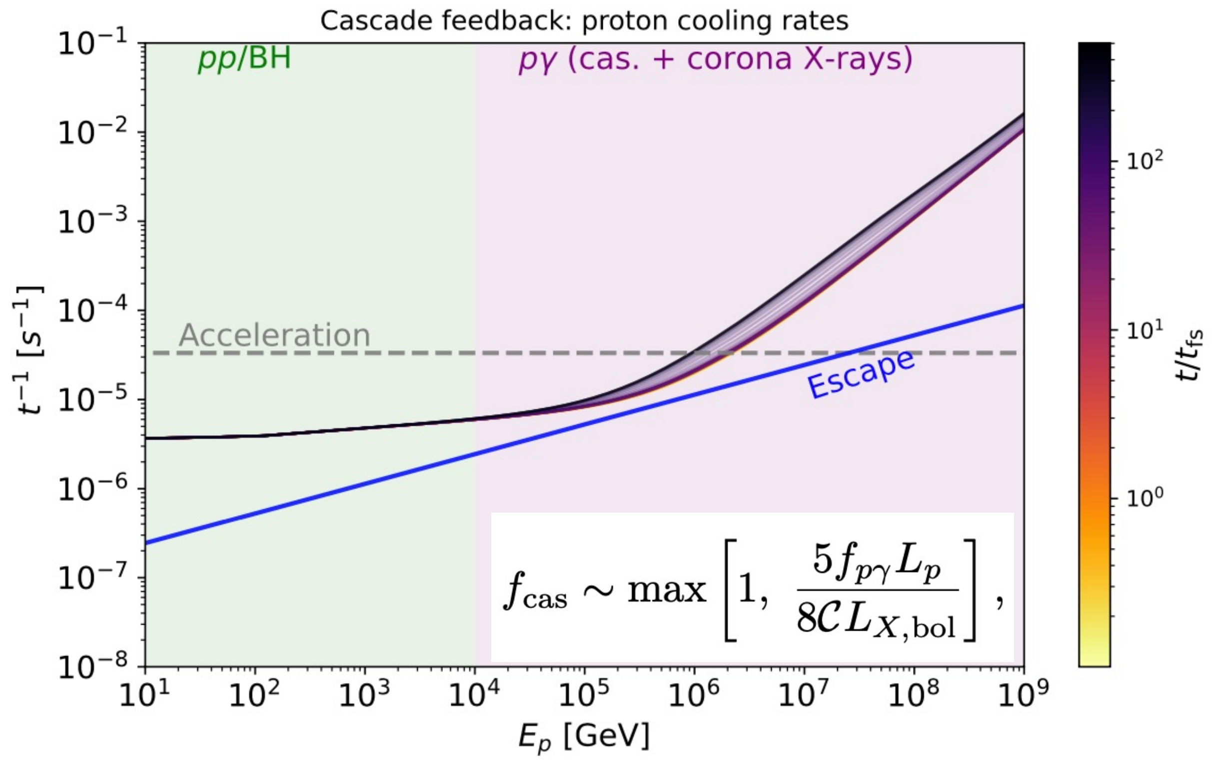


Yuan et al., 2025, arXiv: 2508.08233

- Protons could be accelerated to ~100 TeV, explaining the IceCube neutrino spectra
- In-source $\gamma\gamma$ annihilation depletes γ-rays above 1 MeV, consistent with Fermi observations
- Radiation feedback is subdominant as proton luminosity (Lp) is lower than coronal/disk X-ray luminosity (Lx).

Test the feedback-dominated case





- Use a low disk/coronal X-ray luminosity $L_{X,\rm bol} = 5\times10^{41}\ {\rm erg\ s^{-1}} < L_p \simeq 4\times10^{42}\ {\rm erg\ s^{-1}}$
- EM cascade could enhance the proton cooling rate by a factor of f_cas ~ 2-3
- The proton spectra shift to lower energies
- Radiation feedback is important to dimmer X-ray coronae

Part 3

Transient coronae in tidal disruption events

Tidal disruption events

When a massive star passes close enough to a SMBH

- ~ half of the star's mass remains bounded by the SMBH gravitational force
- Mass accretion -> relativistic jet -> months/year-long flare (optical transient)
- Energy to be reprocessed by accretion ~ 10^{54} erg

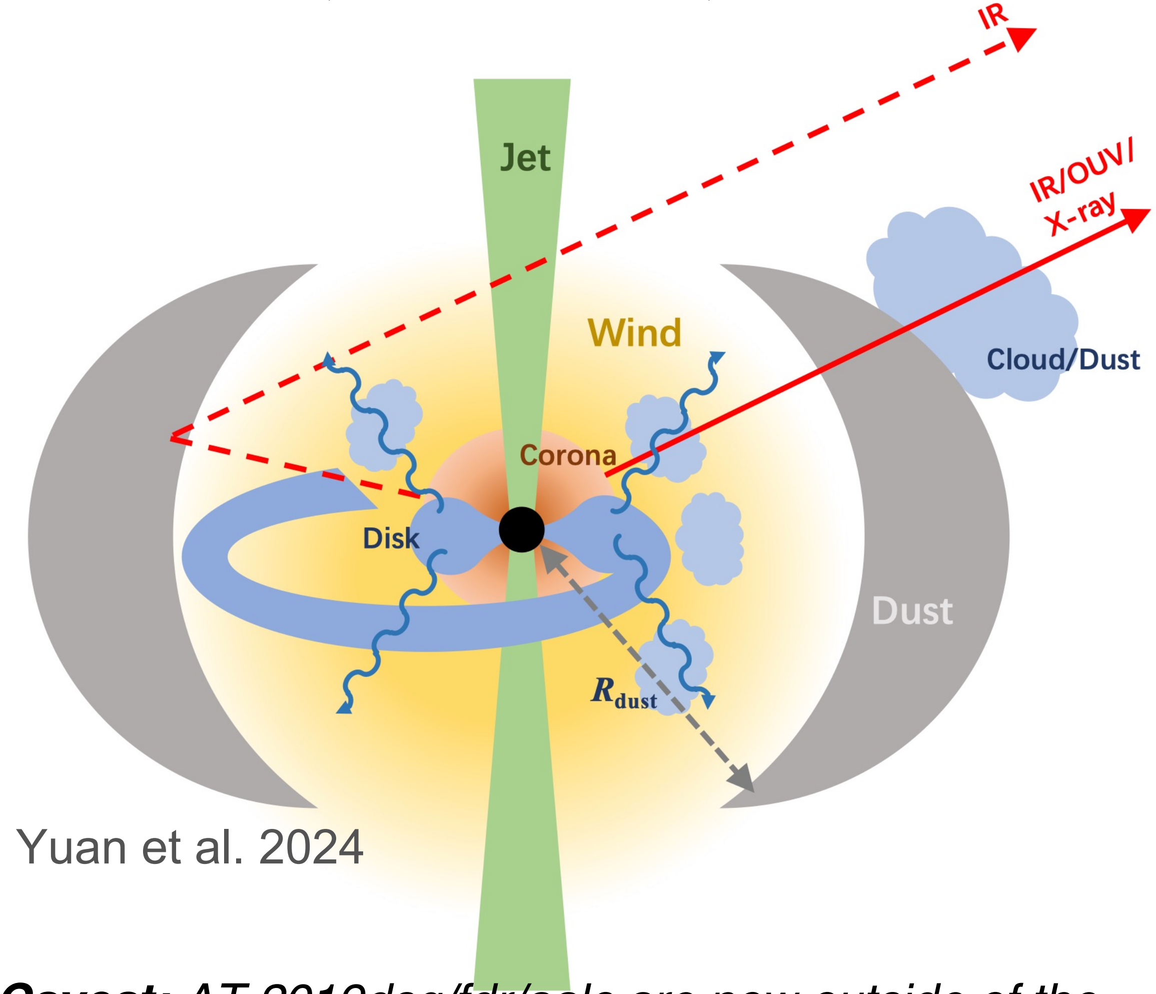
TDE

- Fallback rate $\propto t^{-5/3}$ (Phinney 1989)
- Thermal black body (bb) emissions in optical/UV (OUV) bands
- Some (~1/4) TDEs are observed in (thermal) X-ray and infrared (IR)

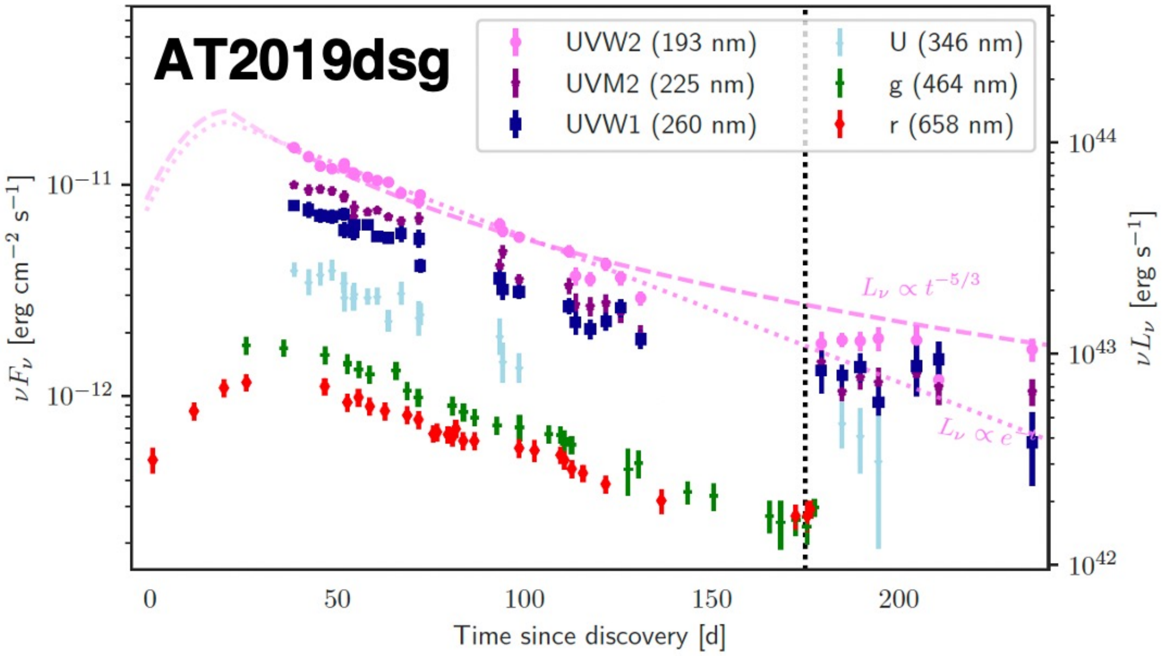
Video credit: DESY Science communication lab

7 TDEs are possibly associated with neutrinos

Stein et al. 2021; Reusch et el. 2021; van Velzen et al. 2021; Jiang et al. 2023; Yuan et al. 2024; Li et al. 2024



Yuan et al. 2024



Possible sites for particle acceleration:
Jet, disk, stream collision, wind, corona (?)

Caveat: *AT 2019dsg/fdr/aalc are now outside of the 90% uncertainty region of corresponding neutrino events in updated IceCube catalog IceCat-2*

(Wang + 11; Dai + 15; Wang & Liu 16; Dai & Fang 17; Lunardini & Winter 17; Senno + 17; Hayashaki & Yamazaki 19; Fang 20; Murase+ 20; Winter & Lunardini, 2023; Yuan & Winter, 2023; Yuan +, 2025)

Transient TDE coronae: a phenomenological scenario

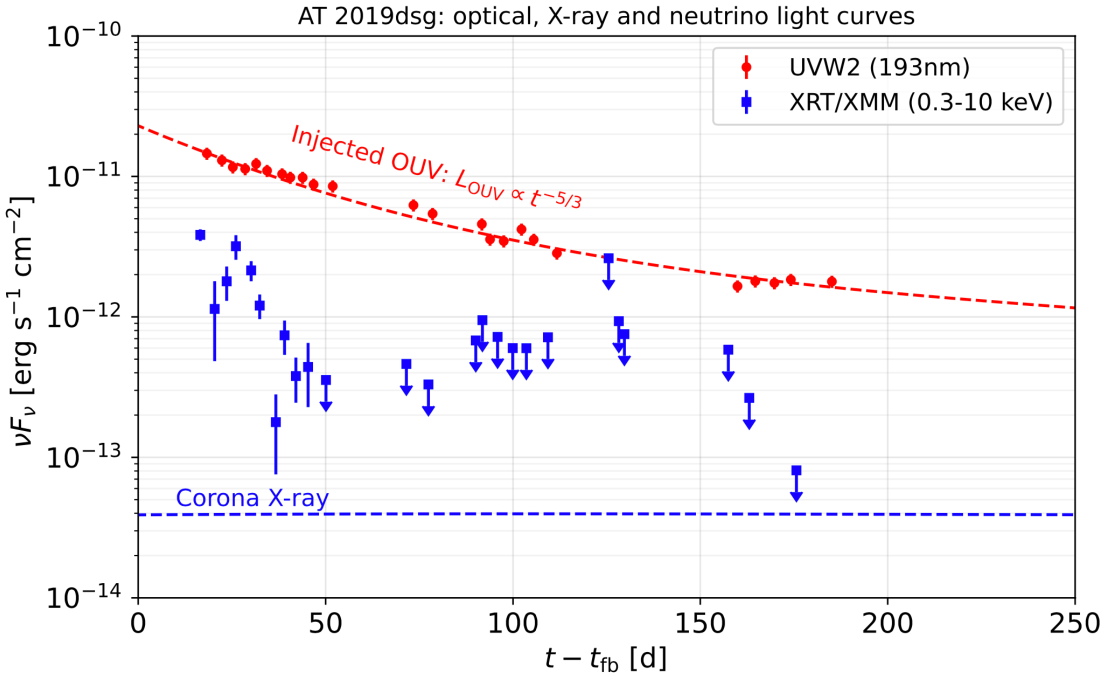
Assumption: ***decaying accretion rate lead to decaying particle density and Thomson opacity***

$$\tau_T \;=\; \tau_0 \min[1, \dot{M}/\dot{M}_{\rm Edd}]\;, \quad \dot{M} = \frac{\eta_{\rm acc} M_\star}{3 t_{\rm fb} c^2}\left(\frac{t}{t_{\rm fb}}\right)^{-5/3}, \quad t_{\rm fb} \simeq 3.9\times 10^6 M_7 (M_\star/M_\odot)^{-1/10}\ {\rm s}$$

($\tau_0 = 0.5$ as for NGC 1068, $\dot{M}_{\rm Edd}$ is the Eddington accretion rate, $\eta_{\rm acc} \sim 0.01$ is the accretion efficiency, $M_\star$ is the mass of disrupted star, $t_{\rm fb}$ is the mass fallback time)

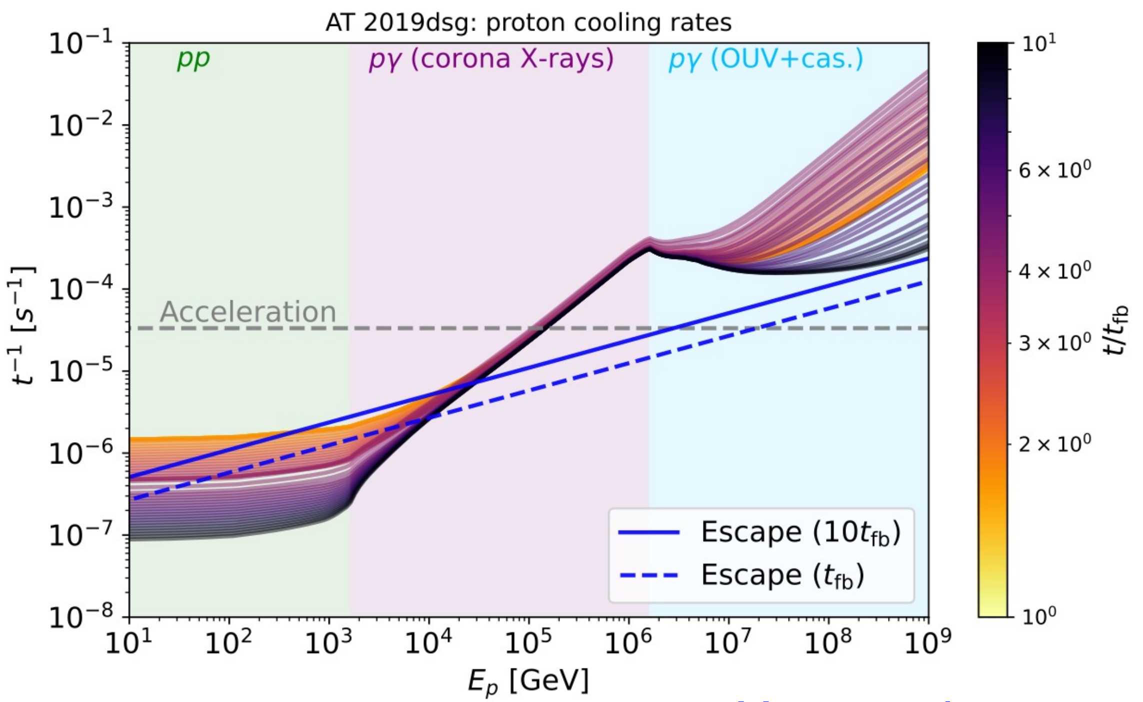
- TDE coronae are typically weaker than AGN cases.
- Proton injection power follows accretion rate
- Steady state is not guaranteed
- X-ray fluxes are used as upper limit of corona X-ray luminosity
- Thermal optical/UV (OUV) emissions are produced at a much larger radius (>> R_co)

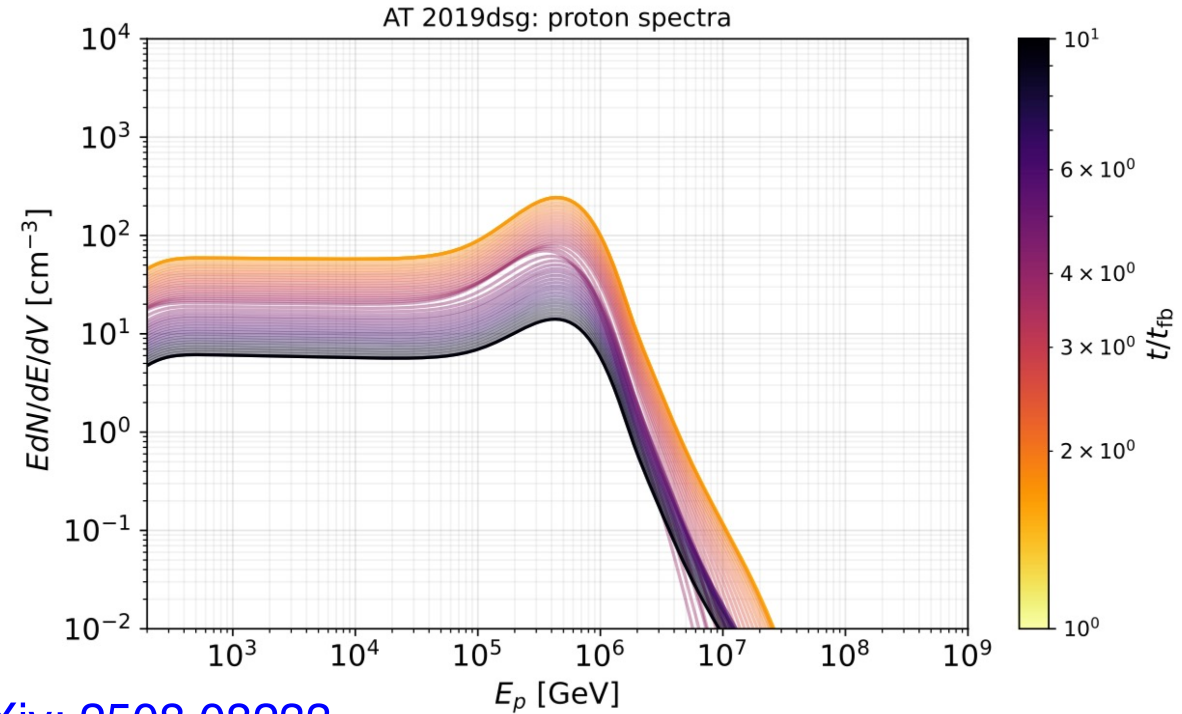
$$R_{\rm OUV} \approx [3GM\dot{M}L_{\rm OUV}/(8\pi\eta_{\rm rad}c^2\sigma_S T^4)]^{1/3}$$



Transient TDE coronae: cooling rates and proton spectra

Run the simulation for coupled acceleration-radiation scenario from 10 t_fb



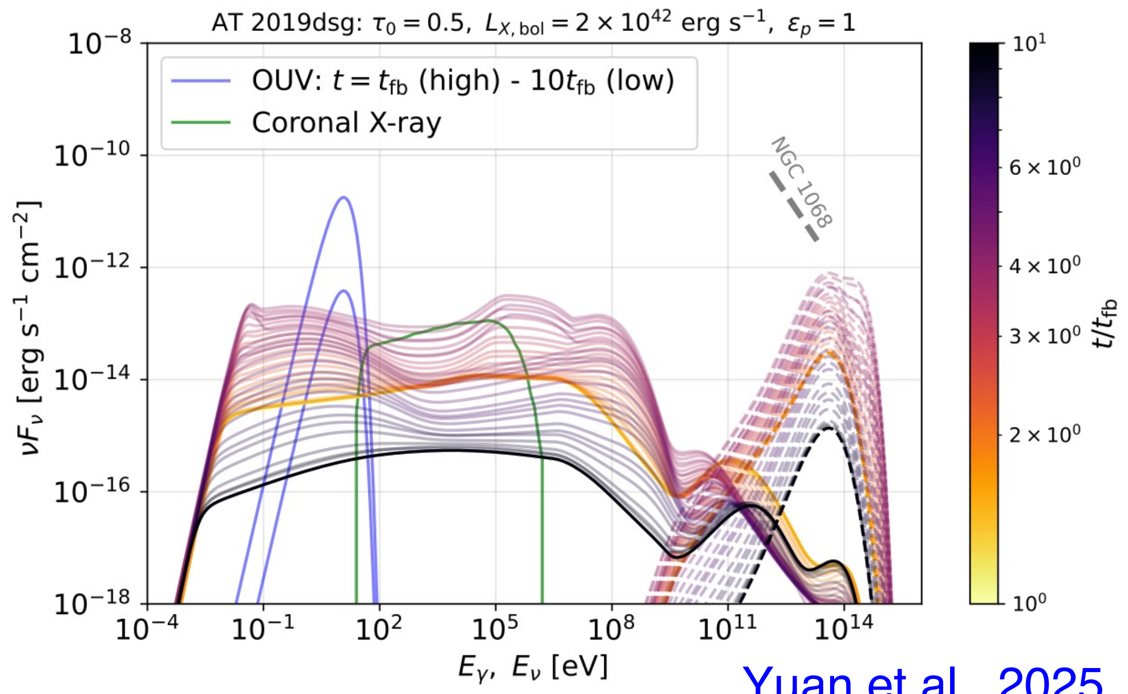


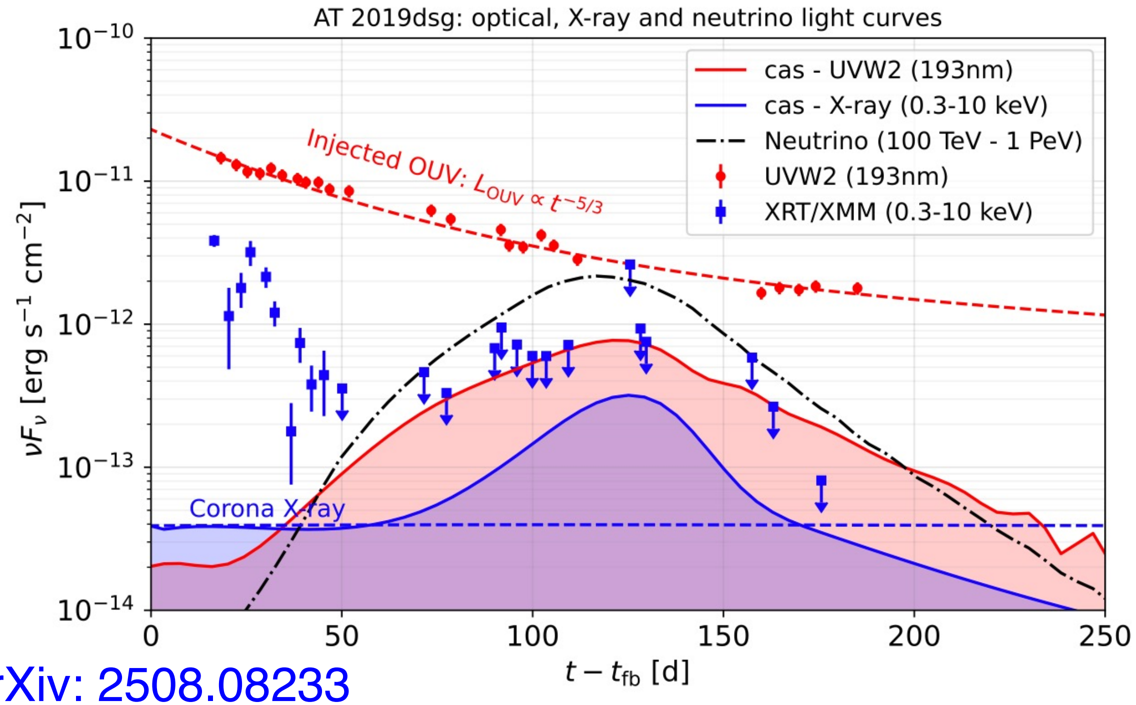
[Yuan et al., 2025, arXiv: 2508.08233]

- Radiation feedback dominates *early stage (super-Eddington)* proton cooling and spectral evolution.
- Proton density initially increases due to the accumulation of accelerated protons for *t < t_esc*
- As accretion rate drops, cooling rates converge to the case driven by coronal X-ray and disk OUV photons.

Transient TDE coronae: SED and lightcurves

Run the simulation for coupled acceleration-radiation scenario to 10 t_fb





Yuan et al., 2025, arXiv: 2508.08233

- Neutrino peak energy ~10-100 TeV; gamma-ray spectra extends to 1 GeV due to weaker $\gamma\gamma$ annihi.
- Delayed coronal OUV, X-ray and neutrino emissions are caused jointly by: *accumulation of protons (t<t_esc); radiation feedback; development of cascades (t_ pγ); duration of super-Eddington phase*
- Testable for subpopulations with potential neutrino correlations or strong non-jetted X-ray emissions, as the coronal contribution would be prominent.

Summary and outlook

- This framework efficiently solves the *time-dependent* Fokker–Planck equations for proton acceleration, self-consistently coupled to a leptonic–hadronic radiation model, reconciling the tiny time steps in compact acceleration regions with the long-term evolution of the system.

- For NGC 1068, this code self-consistently models the neutrino and EM cascade spectra from a steady-state corona, which explains the neutrino observations.

- For TDEs, where the X-ray emission from the corona is typically weak, EM cascade feedback can be more important. A transient corona scenario predicts delayed OUV, X- ray, and neutrino emissions from early-stage EM cascade feedback.

- The steady corona model can be directly applied to other neutrino-emitting Seyfert galaxies, such as NGC 4151, NGC 3079, NGC 7469, and the Circinus galaxy, whereas the transient corona model is testable via multi-messenger observations of TDEs.

- The code's flexible timescales and injection terms enable modeling of other mechanisms such as magnetic reconnection, shear flow acceleration, and shock acceleration.

Thank you for your attention!

Backup Slides

AM^3 : solver optimization

Numerically solving the coupled PDEs for electron, proton, neutrons, neutrino and photon distributions.

$$\partial_t n_i = Q_{i,ext} + \sum_k Q_{int,k\to i} - \partial_E(\dot{E}\cdot n_i) - (\alpha_{i,esc} + \alpha_{i,adv})n_i$$

Injection   Cooling   Escape/Advection

- An ***Open-Source*** Tool for ***Time-Dependent*** Lepto-Hadronic Modeling of Astrophysical Sources
- Blazars, GRBs, TDEs, etc

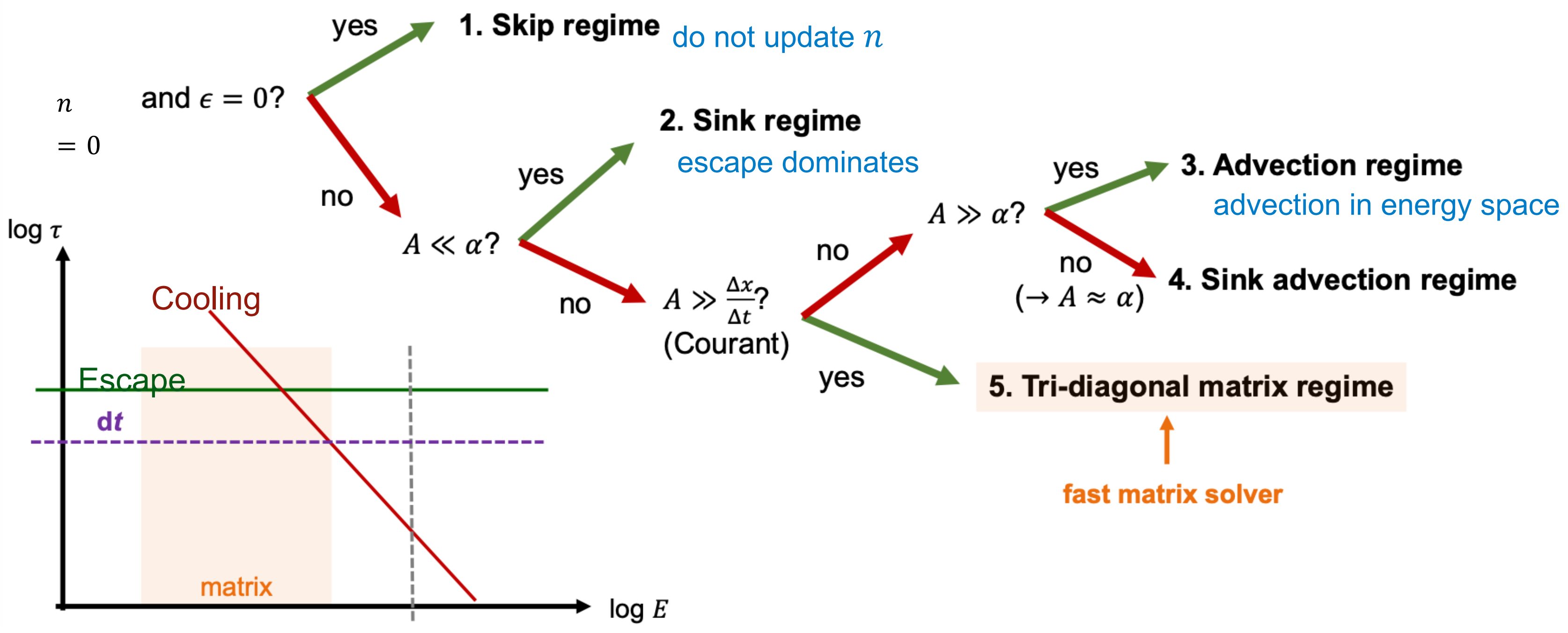
(Klinger, Rudolph, Rodrigues, CY +, arXiv: 2312.13371, ApJS)

Solver optimization

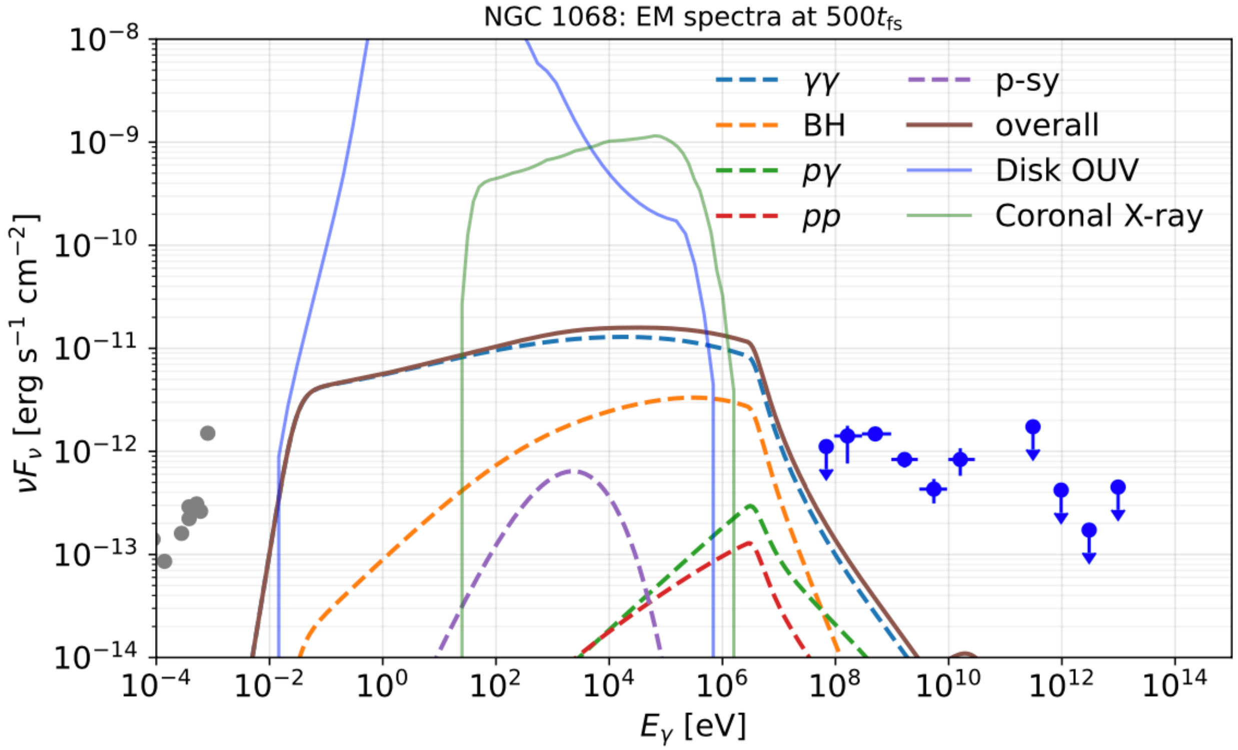
- dimensionless $x = \ln(E/E_0)$
- cooling term $A = \dot{E}/E$,
- injection $\epsilon = EQ(E)$
- escape α

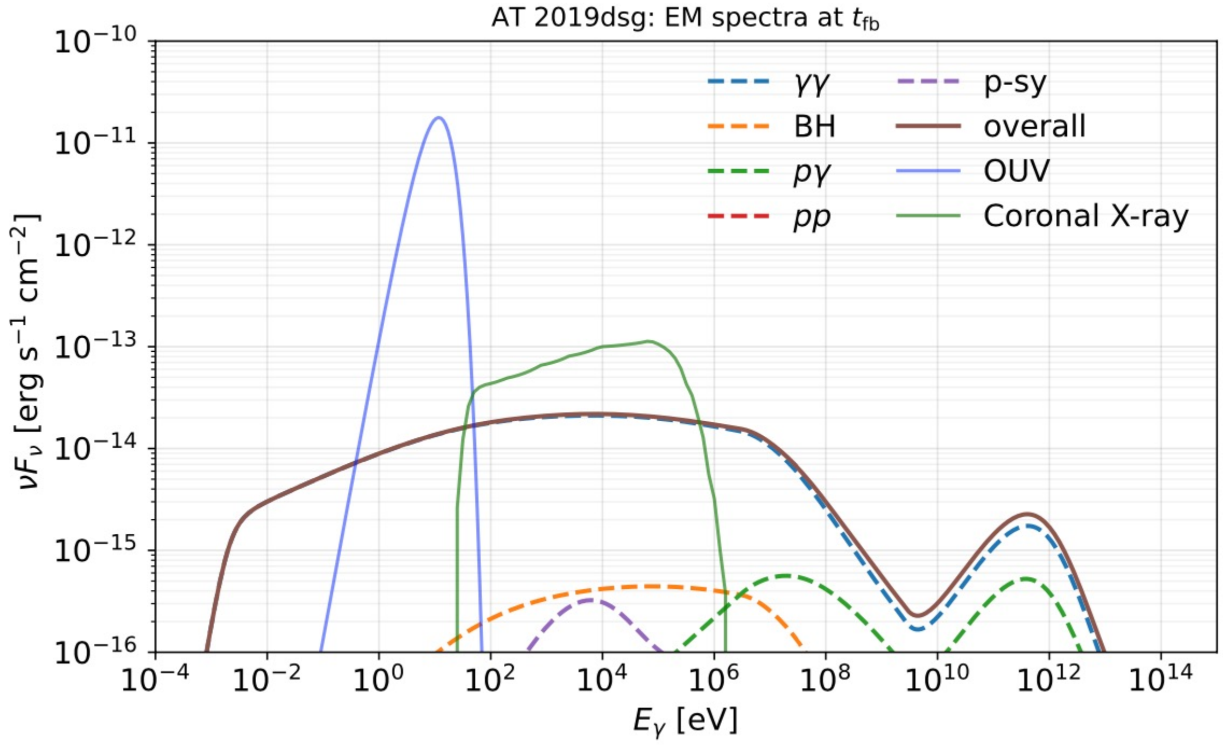
Courant–Friedrichs–Lewy stability criterion:

- Full treatment is needed if $A \gg \Delta x/\Delta t$.
- force the time step Δt to be driven by the smallest timescales

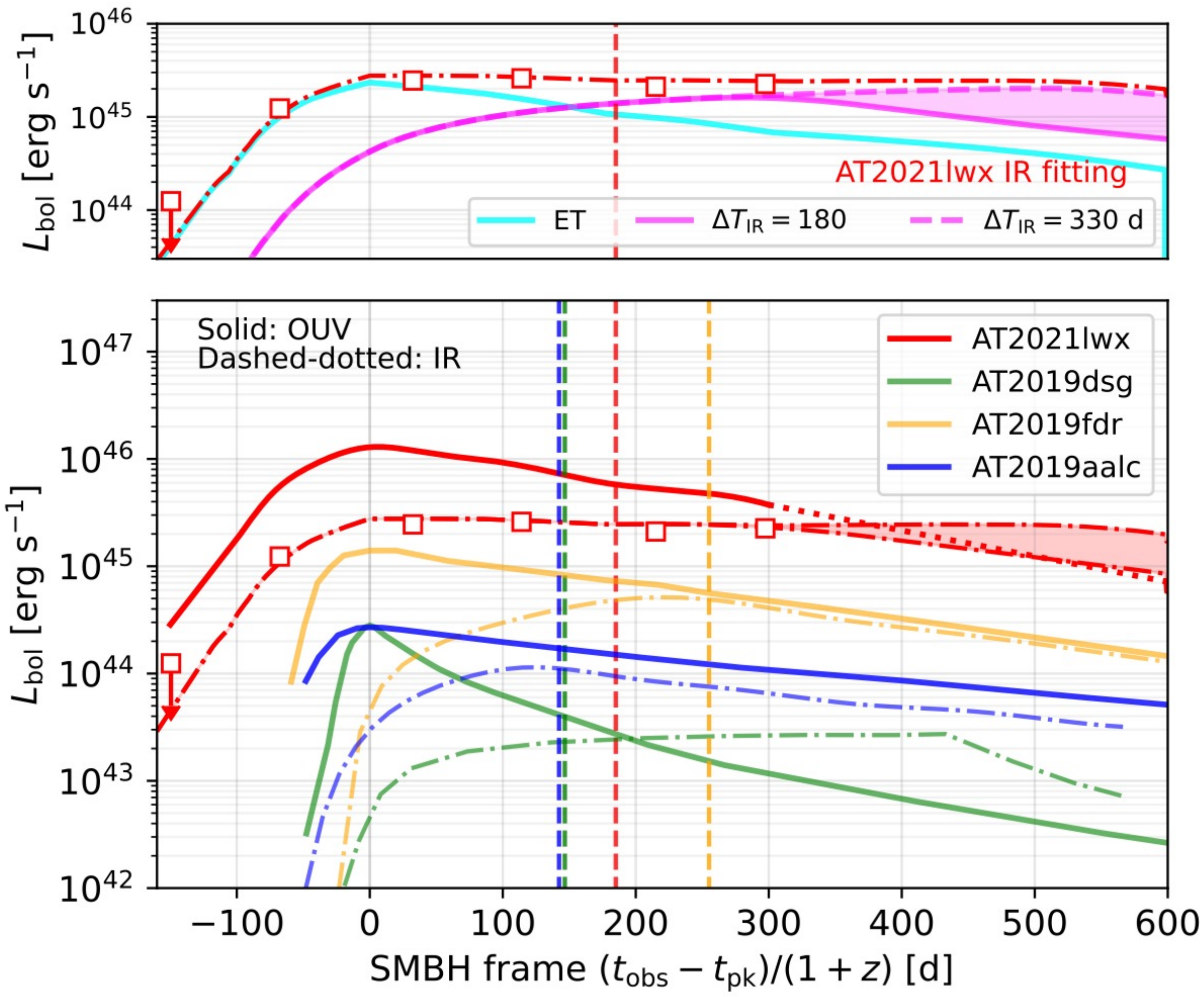


Spectral components





TDE similarities: IR echoes and delayed neutrino signals



CR composition

Composition of accelerated particles

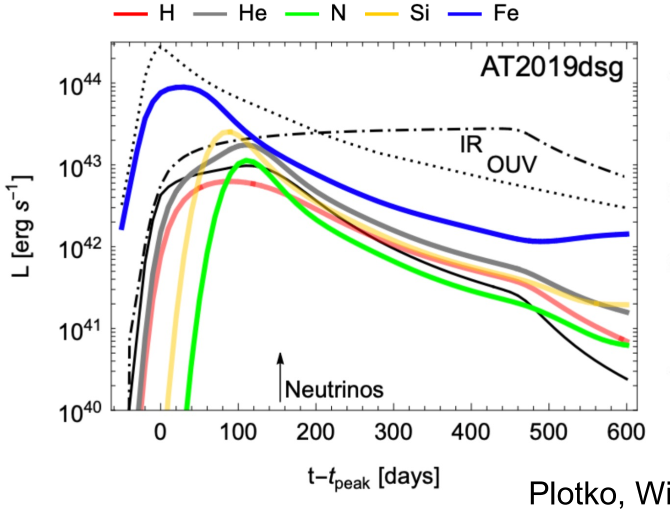
Theoretical simplification: Proton-dominant injection

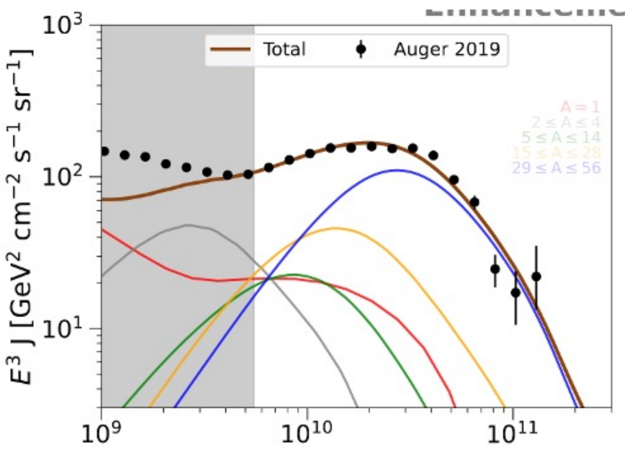
Real physics:

- Injection of heavy nuclei (He, CNO, Na, Si, Fe) and isotopes
- Photon disintegration in radiation zones and during propagating to the Earth
- Consider the nuclear cascade (j -> i)

$$Q_i = Q_{i,\mathrm{ext}} + \sum_j \int dE\, N_j(E)\Gamma_j(E)\frac{dn_{j\to i}}{dE}(E, E_i)$$

- One example: UHECRs from TDE winds





Plotko, Winter, Cecilia, CY (2024)

Particle injection spectrum

Theoretical simplification:

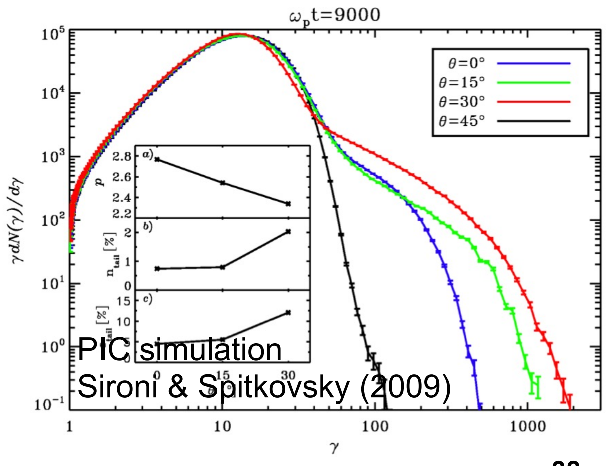
- power-law with an exponential cutoff

$$dn/dE \propto E^{-s}\exp(-E/E_{\max}),\ E > E_{\min}$$

Real physics:

- low-energy thermal tails
- modified spectra from different acceleration mechanisms (e.g., magnetic reconnection in high-magnetized zones, plasma turbulence) -> *MHD simulations*

Applications:
AGN/TDE disk corona
GRB internal shocks
Pulsar wind nebulae
…



PIC simulation
Sironi & Spitkovsky (2009)

Numerically solve the FP equation

Supplemental material of arXiv: 2508.08233

$$\frac{\partial f}{\partial t} = \frac{1}{p^2}\frac{\partial}{\partial p}\left[p^2 D_p(p)\frac{\partial f}{\partial p} + \frac{p^3}{t_{\rm cool}} f\right] - \frac{f}{t_{\rm esc}} + q(p),$$

$t_n = t_0 + n\Delta t$ (for $n = 1, 2, \ldots$) and $p_i = p_0\,(p_{\rm max}/p_0)^{i/N}$

- $f_i^n \equiv f(p_i, t_n)$, $D_i \equiv D_p(p_i)$, $\dot{p}_i \equiv \dot{p}(p_i)$, $q_i \equiv q(p_i)$, $\lambda_i \equiv 1/t_{\rm esc}(p_i)$
- $x_{i+1/2} = (x_i + x_{i+1})/2$, $\Delta x_{i+1/2} = x_{i+1} - x_i$, $\Delta x_i = (x_{i+1} - x_{i-1})/2$ for $x = D_p$, p and $\dot{p}$. For instance, $p_{i+1/2} = (p_i + p_{i+1})/2$.

The flux term $\Phi(p,t) \equiv D_p(p)\frac{\partial f}{\partial p} + \dot{p} f$ could be discretized as

$$\Phi^n_{i+1/2} = D_{i+1/2}\frac{f^n_{i+1} - f^n_i}{\Delta p_{i+1/2}} + \dot{p}_{i+1/2}[(1-\delta) f^n_{i+1} + \delta f^n_i],$$

where $0 \le \delta \le 1$ is the Chang-Cooper weighting factor [1] defined as

$$\delta = \frac{1}{w} - \frac{1}{e^w - 1}$$

with $w = \Delta p_{i+1/2}\dot{p}_{i+1/2}/D_{i+1/2}$. This method ensures both the preservation of positivity and the correct equilibrium solution, even in strongly cooling-dominated ($w \gg 1$) regimes. In practice, we find that $w \gg 1$, especially at high p, where proton cooling dominates the spectral evolution and the resulting $\delta \to 0$, corresponding to the upwinding scheme. The physical meaning is that when the cooling rate is exceedingly high, protons flow from high p_{i+1} to low p_i.

Numerically solve the FP equation

Supplemental material of arXiv: 2508.08233

This implicit scheme is stable for linear problems while maintaining second-order accuracy in time, and it can be expressed in a tridiagonal matrix form as

$$A_i f_{i-1}^{n+1} + B_i f_i^{n+1} + C_i f_{i+1}^{n+1} = R_i^n .$$

The system can efficiently evolve from t_n to t_{n+1} by applying the inverse of a tridiagonal matrix $\mathcal{M}$,

$$(\vec{f}_i^{n+1})^T = \mathcal{M}^{-1} \cdot (\vec{R}_i^n)^T, \tag{3}$$

where $\vec{f}_i^{n+1} = (f_0^{n+1}, ..., f_N^{n+1})$, $\vec{R}_i^n = (R_0^n, ..., R_N^n)$, and $\mathcal{M}$ can be explicitly written as

$$\mathcal{M} = \begin{bmatrix} B_0 & C_0 & 0 & 0 & \cdots & 0 \\ A_1 & B_1 & C_1 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & A_{N-1} & B_{N-1} & C_{N-1} \\ 0 & \cdots & 0 & 0 & A_N & B_N \end{bmatrix} . \tag{4}$$

By repeating the above procedure K times and updating the input terms accordingly, the proton distribution at time $t_K = t_0 + K\Delta t$ can be obtained.